QCD resummation for jet and hadron production

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Outline:

- Introduction: QCD threshold resummation
- Drell-Yan process
- Resummation in QCD hard-scattering
- Hadron pair production in pp collisions
- Jet production at the LHC

Focus on phenomenology, less on technical aspects of resummation

Introduction:

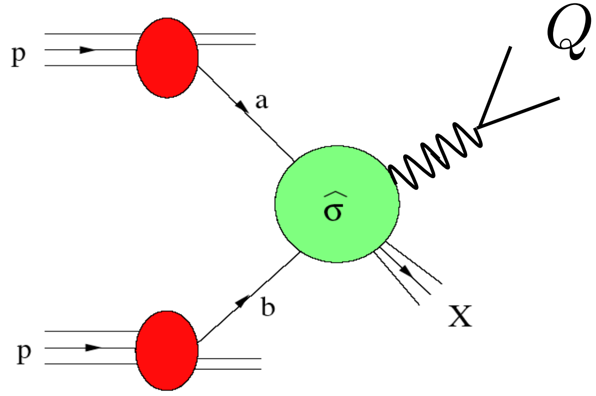
QCD threshold resummation

Hard-scattering reactions play central role in QCD:

- Probes of nucleon structure
- Involved in most of today's hadron collider physics ("New Physics", heavy ions, polarized protons…)
- Test our understanding of QCD at high energies, and our ability to do "first-principles" computations

Cornerstones: factorization & asymptotic freedom

Factorized cross section:
e.g. Drell-Yan

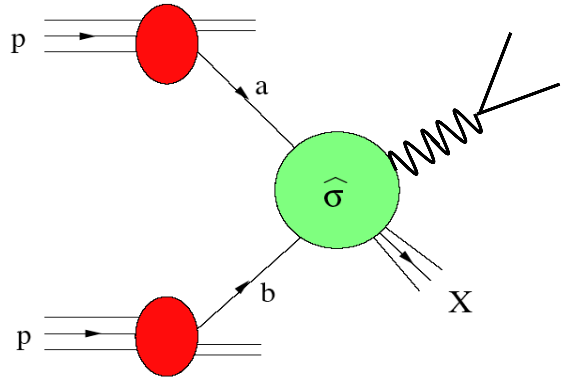


$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a, \mu) \, f_b(x_b, \mu) \, \omega_{ab} \left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu} \right) \; + \; \ldots$$

- $f_{a,b}$ parton distributions: non-pert., but universal
- ω_{ab} partonic cross sections: process-dep., but pQCD

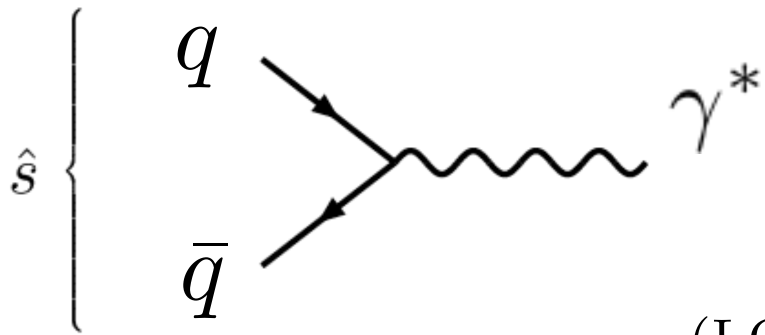
$$\omega_{ab} \; = \; \omega_{ab}^{(\mathrm{LO})} \; + \; \frac{\alpha_s}{2\pi} \, \omega_{ab}^{(\mathrm{NLO})} \; + \; \ldots$$

- $\mu \sim Q$ factorization / renormalization scale
- corrections power-suppressed in Q



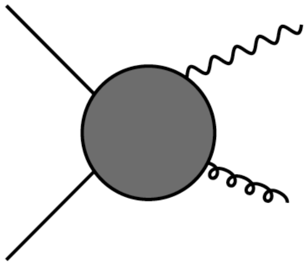
$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu)\, f_b(x_b,\mu)\, \omega_{ab}\left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu}\right) + \ldots$$

LO :



$$\omega_{ab}^{(\mathrm{LO})} \propto \delta(1-z)$$

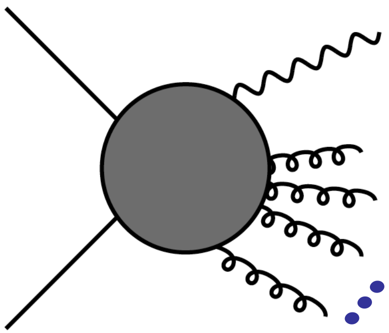
- NLO correction:



$$z \to 1:$$

$$\omega_{ab}^{(\mathrm{NLO})} \propto \alpha_s \left(\frac{\log(1-z)}{1-z}\right)_+ + \ldots$$

- higher orders:



$$\omega_{ab}^{(\mathrm{N^kLO})} \propto \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z}\right)_+ + \ldots$$

"threshold logarithms"

- for z → 1 real radiation inhibited

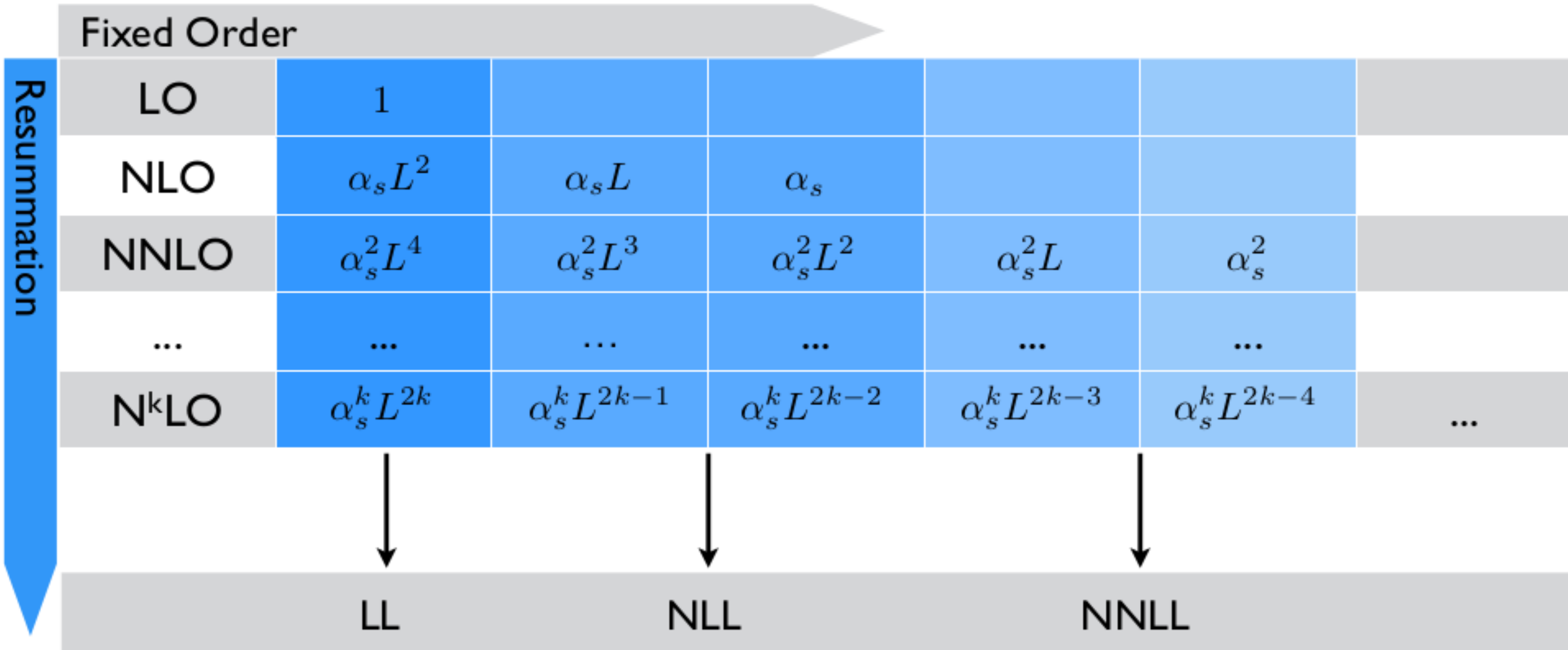
- logs emphasized by parton distributions :

$$d\sigma \sim \int_\tau^1 \frac{dz}{z} \mathcal{L}_{q\bar{q}}\left(\frac{\tau}{z}\right) \omega_{q\bar{q}}(z) \qquad \tau = \frac{Q^2}{S}$$

z = 1 relevant,
in particular as $\tau \rightarrow 1$

Large logs can be resummed to all orders

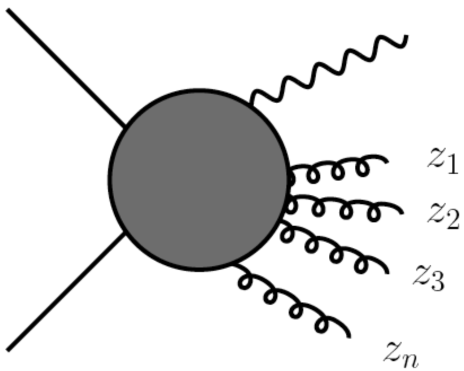
Catani, Trentadue; Sterman; …



- factorization of matrix elements

$$|M|^2(p,\bar{p};k_1,\ldots,k_n) \sim \frac{1}{n!}\left[\prod_{i=1}^{n}\frac{(p\cdot\bar{p})}{(p\cdot k_i)(\bar{p}\cdot k_i)}\right]|M|^2_{\mathrm{LO}}$$

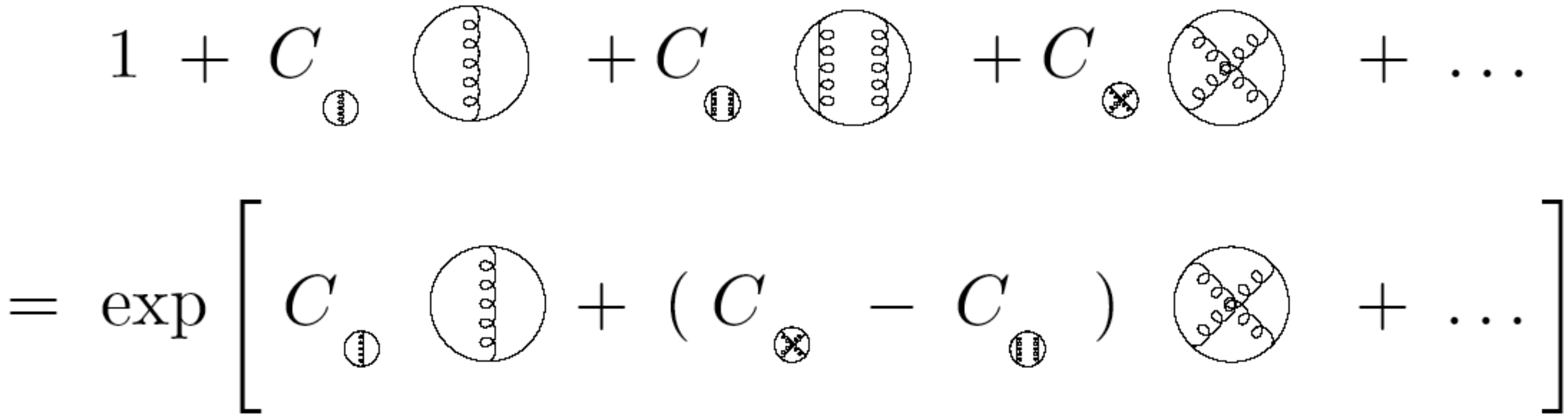
- …and of phase space when integral transform is taken:



$$\delta\left(1-z-\sum_{i=1}^{n}z_i\right) = \frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N\left(1-z-\sum_{i=1}^{n}z_i\right)}$$

$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

- exponentiation: Gatherall; Franklin, Taylor; Sterman



$\alpha_s^k L^{2k}$ $\qquad$ $\alpha_s^k L^{2k-1}$

$$1+\alpha_s L^2+\alpha_s^2 L^4+\ldots+\alpha_s L+\alpha_s^2 L^3+\ldots$$

$$\leftrightarrow \exp\left[\alpha_s L^2+\alpha_s^2 L^3+\ldots+\alpha_s L+\alpha_s^2 L^2+\ldots\right]$$

$\alpha_s^k L^{k+1}$ $\qquad$ $\alpha_s^k L^k$

$\overline{\text{MS}}$ scheme

$$\hat{\sigma}^{\rm res}_{q\bar{q}}(N) \propto \exp\left[2\int_0^1 dy\, \frac{y^N - 1}{1-y} \int_{\mu^2}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) + \ldots \right]$$

$$A_q(\alpha_s) = C_F \left\{ \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{C_A}{2}\left(\frac{67}{18} - \zeta(2)\right) - \frac{5}{9} T_R n_f \right] + \ldots \right\}$$

LL :

$$\tilde{\omega}^{(\rm res)}_{q\bar{q}}(N) \propto \exp\left[+ \frac{2C_F}{\pi} \alpha_s \ln^2 N + \ldots \right]$$

- threshold logs enhance cross section

proper expansion:

Catani, Mangano, Nason, Trentadue

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \propto \exp\left\{2\ln\bar{N}\, h^{(1)}(\lambda) + 2h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right)\right\}$$

LL NLL

$$\lambda = \alpha_s(\mu^2)\, b_0\, \log(N e^{\gamma_E})$$

$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right]$$

$$h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right) = \ldots$$

Inverse transform:

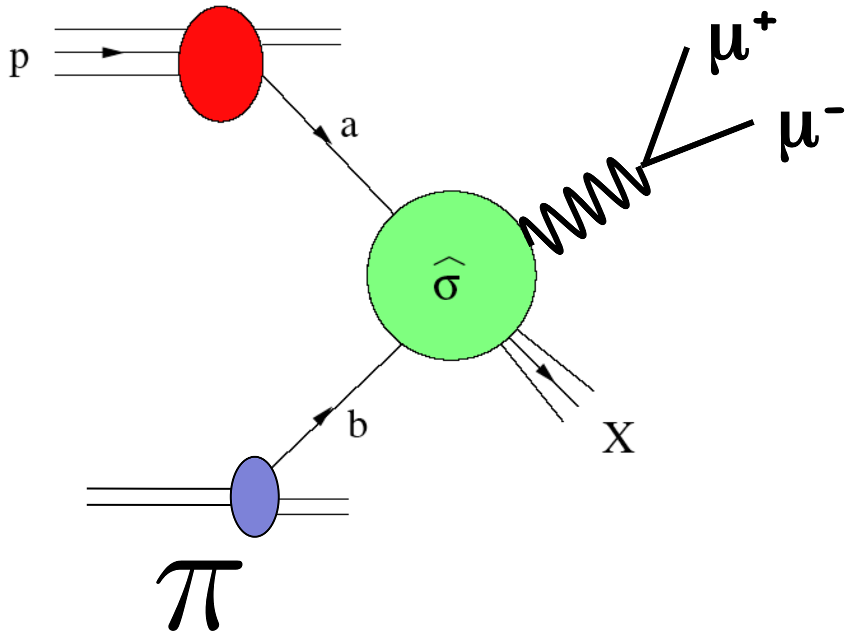
$$\sigma^{\mathrm{res}} = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dN\, \tau^{-N}\, \tilde{\sigma}^{\mathrm{res}}(N)$$

Drell-Yan process in πN scattering

M. Aicher, A.Schäfer, WV

- Drell-Yan process has been main source of information on pion structure:

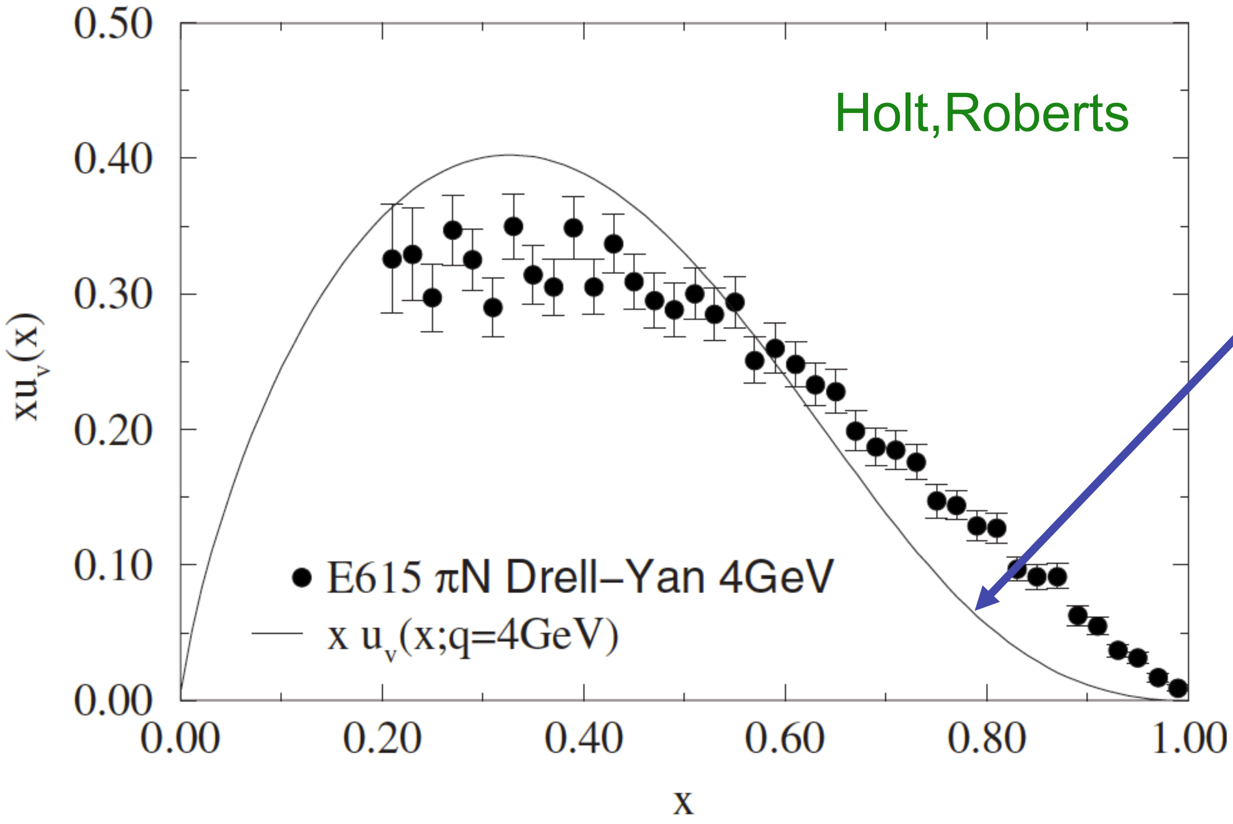
E615, NA10



$$d\sigma = \sum_{ab} \int dx_a \int dx_b \, f_a^{\pi}(x_a, \mu) f_b(x_b, \mu) \, d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu)$$

- Kinematics such that data mostly probe valence region: ~200 GeV pion beam on fixed target

- LO extraction of u_v from E615 data: $\sqrt{S} = 21.75\,\mathrm{GeV}$



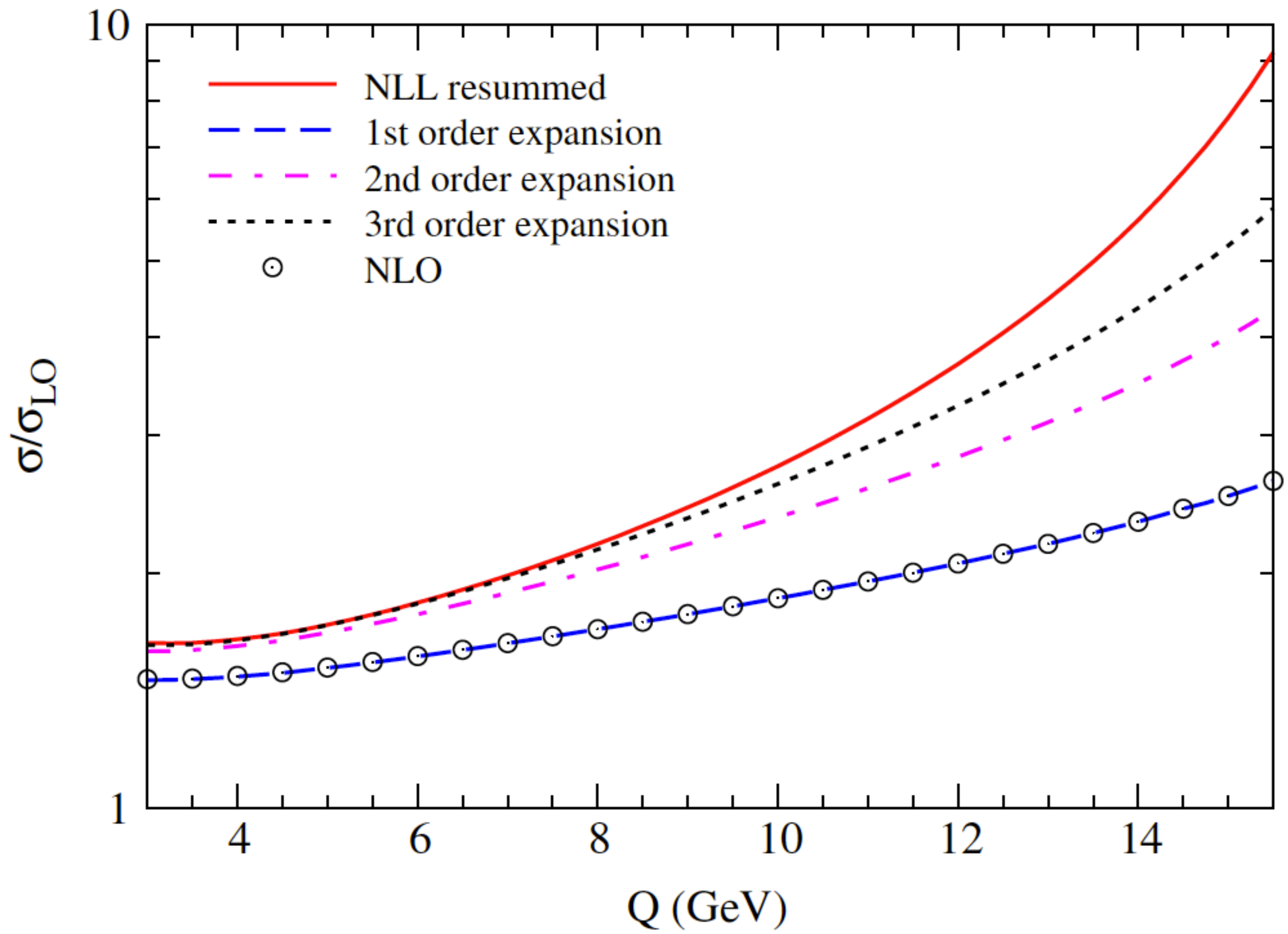
$\sim (1-x)^2$

QCD counting rules

Farrar, Jackson;
Berger, Brodsky; Yuan
Blankenbecler,
Gunion, Nason

Dyson-Schwinger

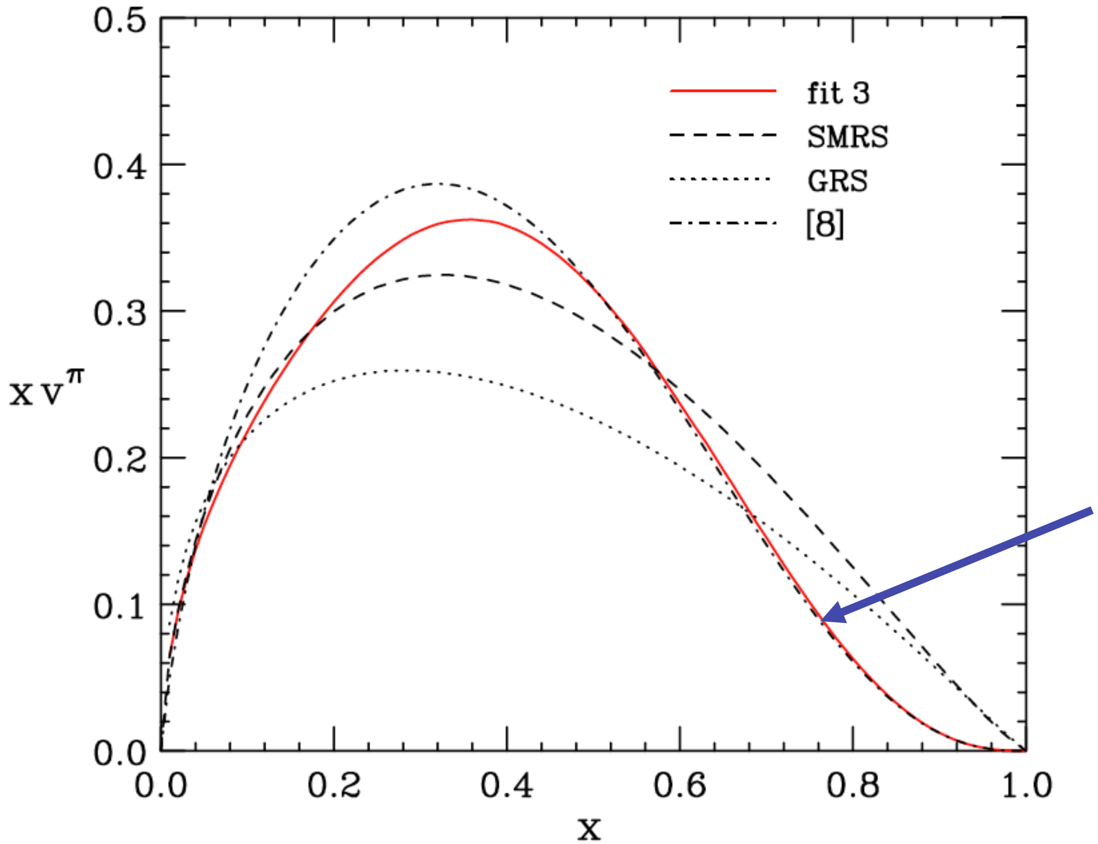
Hecht et al.



Aicher, Schäfer, WV

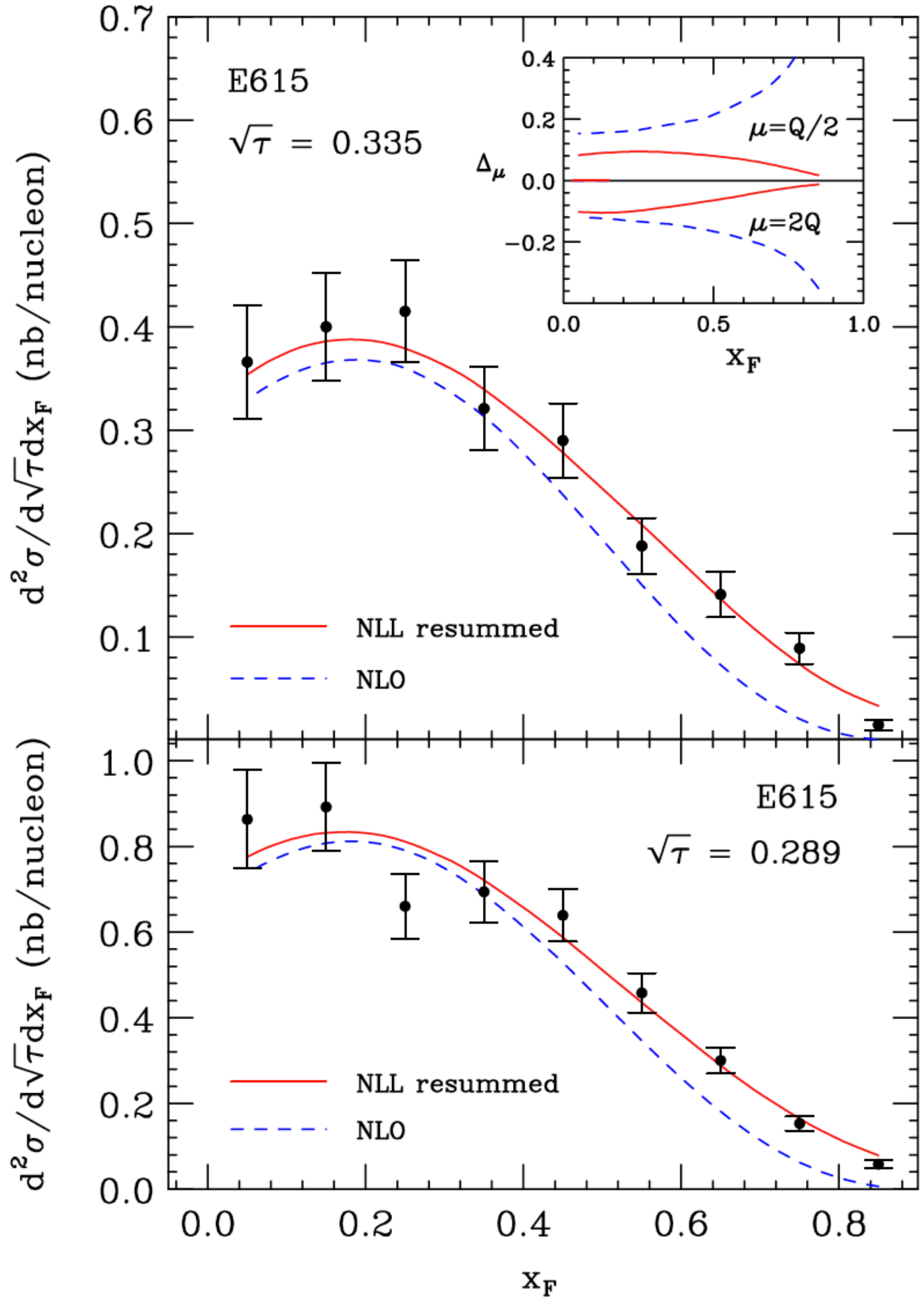
$$xv^{\pi}(x, Q_0^2) = N_v x^{\alpha}(1-x)^{\beta}(1+\gamma x^{\delta})$$

Fit	$2\langle xv^{\pi}\rangle$	α	β	γ	K	χ^2 (no. of points)
1	0.55	0.15 ± 0.04	1.75 ± 0.04	89.4	0.999 ± 0.011	82.8 (70)
2	0.60	0.44 ± 0.07	1.93 ± 0.03	25.5	0.968 ± 0.011	80.9 (70)
3	0.65	0.70 ± 0.07	2.03 ± 0.06	13.8	0.919 ± 0.009	80.1 (70)
4	0.7	1.06 ± 0.05	2.12 ± 0.06	6.7	0.868 ± 0.009	81.0 (70)



$Q = 4\ \mathrm{GeV}$

$\sim (1-x)^{2.34}$



E615
$\sqrt{\tau}$ = 0.335
Δ_μ
μ=Q/2
μ=2Q
x_F
NLL resummed
NLO
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)
E615
$\sqrt{\tau}$ = 0.289
NLL resummed
NLO
x_F

Resummation in QCD hard-scattering

- Color singlet hard LO scattering $q\bar{q} \rightarrow \gamma^*$

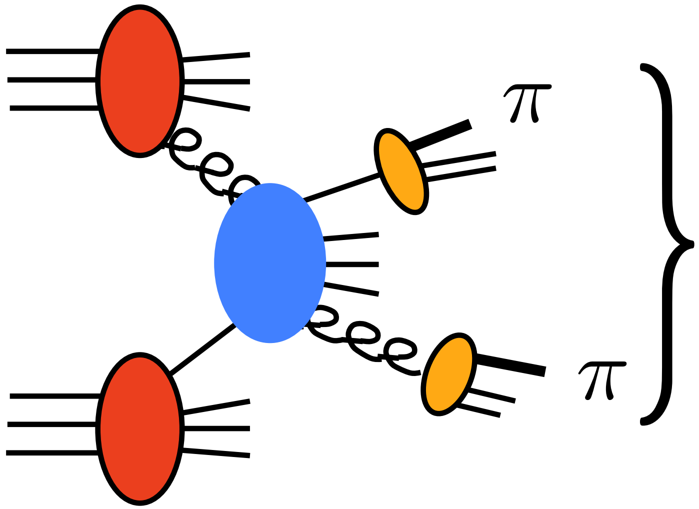
- Natural connection to $gg \rightarrow \mathrm{Higgs}$

- Now: processes with underlying QCD hard scattering:

$$pp \rightarrow \mathrm{hadron(s)} + X$$

$$pp \rightarrow \mathrm{jet} + X$$

- Pair-invariant mass (PIM) kinematics:

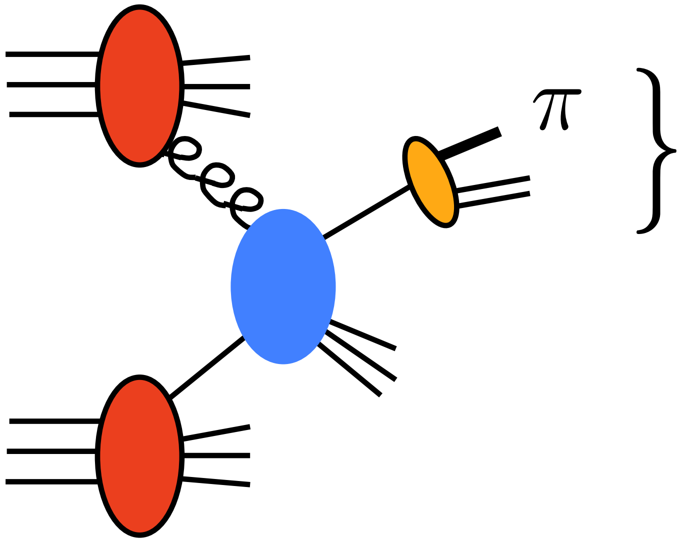


pair mass^2

$$M^2 = (p_\pi + p'_\pi)^2$$

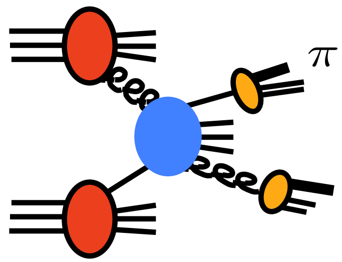
"like" Drell-Yan

- One-particle inclusive (1PI) kinematics:



$$p_T, \eta$$

PIM:



Define $\bar{\eta} = \frac{1}{2}(\eta_1 + \eta_2)$ $\Delta\eta = \frac{1}{2}(\eta_1 - \eta_2)$

$$M^4 \frac{d\sigma^{H_1 H_2 \to h_1 h_2 X}}{dM^2 d\Delta\eta d\bar{\eta}} = \sum_{abcd} \int_0^1 dx_a dx_b\, dz_c\, dz_d\, f_a^{H_1}(x_a) f_b^{H_2}(x_b)\, z_c D_c^{h_1}(z_c) z_d D_d^{h_2}(z_d)$$

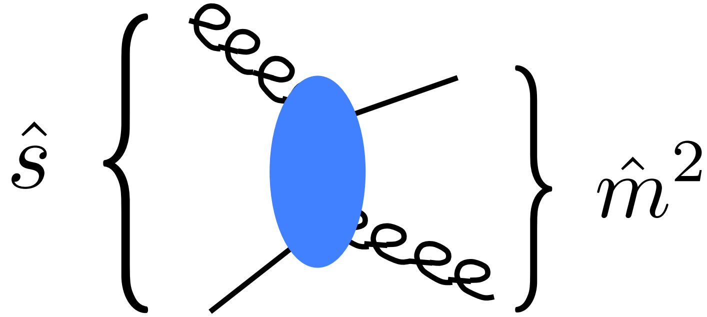
$$\times\, \omega_{ab\to cd}\left(\hat{\tau}, \Delta\eta, \hat{\eta}, \alpha_s(\mu), \frac{\mu}{\hat{m}}\right)$$

with partonic variables

$$\hat{\tau} = \frac{\hat{m}^2}{\hat{s}} \qquad \hat{m}^2 = \frac{M^2}{z_c z_d}$$

$$\hat{\eta} = \bar{\eta} - \frac{1}{2} \ln \frac{x_a}{x_b}$$

LO:

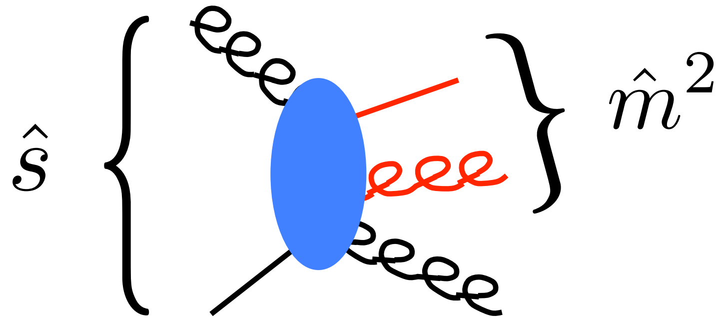


$$\hat{\tau} \equiv \frac{\hat{m}^2}{\hat{s}} = 1$$

$$\omega^{\mathrm{LO}}_{ab\to cd}(\hat{\tau}, \Delta\eta, \hat{\eta}) = \delta(1-\hat{\tau})\,\delta(\hat{\eta})\;\omega^{(0)}_{ab\to cd}(\Delta\eta)$$

cf Drell-Yan

Beyond LO:



e.g. NLO:

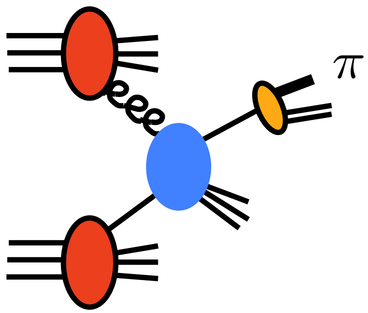
true to all orders!

$$\omega^{\mathrm{NLO}}_{ab\to cd}(\hat\tau,\Delta\eta,\hat\eta) = \delta(\hat\eta)\Big[\omega^{(1,0)}_{ab\to cd}(\Delta\eta)\,\delta(1-\hat\tau) + \omega^{(1,1)}_{ab\to cd}(\Delta\eta)\left(\frac{1}{1-\hat\tau}\right)_+ + \omega^{(1,2)}_{ab\to cd}(\Delta\eta)\left(\frac{\log(1-\hat\tau)}{1-\hat\tau}\right)_+\Big] + \omega^{\mathrm{reg,NLO}}_{ab\to cd}(\hat\tau,\Delta\eta,\hat\eta)$$

at k^th order: threshold logs

$$\alpha_s^k\left(\frac{\log^{2k-1}(1-\hat\tau)}{1-\hat\tau}\right)_+ + \ldots$$

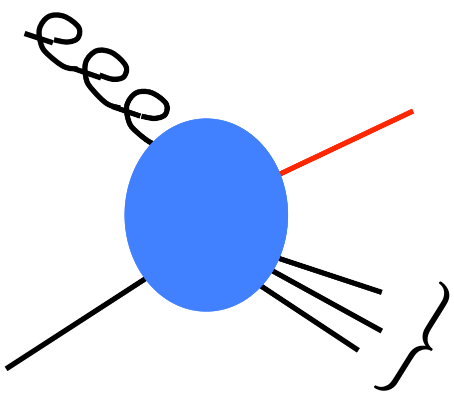
1PI:



$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \sum_{abc} \int_0^1 dx_a\, dx_b\, dz_c\; f_a(x_a)\, f_b(x_b)\, z_c^2 D_c^\pi(z_c)$$

$$\times\, \Omega_{ab\to cX}\left(\hat{x}_T^2, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{\hat{s}}\right)$$

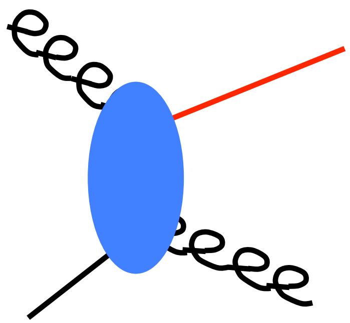
partonic variables: $\hat{x}_T = \dfrac{2p_T}{z_c\sqrt{\hat{s}}}$ $\quad \hat{\eta} = \eta - \dfrac{1}{2}\ln\dfrac{x_a}{x_b}$



mass^2 $\equiv s_4$

$$\hat{x}_T, \hat{\eta} \;\leftrightarrow\; \zeta \equiv 1 - \frac{s_4}{\hat{s}}\;, \hat{\eta}$$

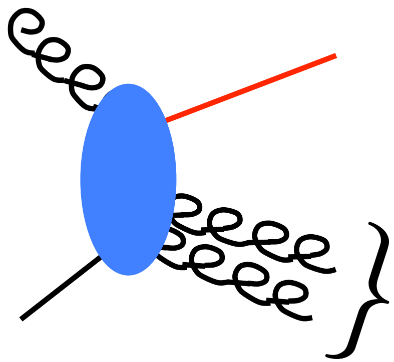
LO:



$$\hat{s}_4 = 0 \qquad \Leftrightarrow \; \zeta = 1$$

$$\Omega^{(\mathrm{LO})}_{ab\to cX}(\zeta,\hat{\eta}) \;=\; \delta(1-\zeta)\,\omega^{(0)}_{ab\to cd}(\hat{\eta})$$

Beyond LO:



$\zeta \neq 1$ not necessarily soft !

at k$^{\mathrm{th}}$ order:

$$\alpha_s^k \left(\frac{\log^{2k-1}(1-\zeta)}{1-\zeta}\right)_+ + \ldots$$

- logs due to soft / collinear emission → resummation
- achieved in Mellin-moment space:

PIM:

$$\int_{-\infty}^{\infty} d\bar{\eta}\, \mathrm{e}^{i\nu\bar{\eta}} \int_0^1 d\tau\, \tau^{N-1} M^4 \frac{d\sigma^{H_1H_2\to h_1h_2X}}{dM^2 d\Delta\eta d\bar{\eta}} \qquad \left(\tau = \frac{M^2}{S}\right)$$

$$= \sum_{abcd} \tilde{f}_a^{H_1}(N+1+i\nu/2)\tilde{f}_b^{H_2}(N+1-i\nu/2)\tilde{D}_c^{h_1}(N+2)\tilde{D}_d^{h_2}(N+2)$$

$$\times \int_{-\infty}^{\infty} d\hat{\eta}\, \mathrm{e}^{i\nu\hat{\eta}} \boxed{\int_0^1 d\hat{\tau}\, \hat{\tau}^{N-1} \quad \omega_{ab\to cd}\left(\hat{\tau}, \Delta\eta, \hat{\eta}, \alpha_s(\mu), \frac{\mu}{\hat{m}}\right)}$$

Likewise, 1PI: moments

$$\int_0^1 d\zeta\, \zeta^{N-1}\, \Omega_{ab\to cX}\left(\zeta, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{\hat{s}}\right)$$

$$\tilde{\omega}^{\rm resum}_{ab\to cd}\left(N,\Delta\eta,\alpha_s(\mu),\frac{\mu}{\hat{m}}\right) = \Delta_a^{N+1}\left(\alpha_s(\mu),\frac{\mu}{\hat{m}}\right)\Delta_b^{N+1}\left(\alpha_s(\mu),\frac{\mu}{\hat{m}}\right)$$

$$\times\,\Delta_c^{N+2}\left(\alpha_s(\mu),\frac{\mu}{\hat{m}}\right)\Delta_d^{N+2}\left(\alpha_s(\mu),\frac{\mu}{\hat{m}}\right)$$

$$\times\,{\rm Tr}\,\{\,H\,S\,\}_{ab\to cd}\left(N,\Delta\eta,\alpha_s(\mu),\frac{\mu}{\hat{m}}\right)$$

soft & coll. gluons

large-angle soft gluons

$$\ln\Delta_i^N\left(\alpha_s(\mu),\frac{\mu}{\hat{m}}\right) = \int_0^1\frac{z^{N-1}-1}{1-z}\int_{\hat{m}^2}^{(1-z)^2\hat{m}^2}\frac{dq^2}{q^2}A_i(\alpha_s(q^2))$$

(like Drell-Yan)

${\rm Tr}\,\{\,H\,S\,\}_{ab\to cd}$ matrix problem in color space

Kidonakis,Oderda,Sterman
Bonciani,Catani,Mangano,Nason
Almeida,Sterman,WV

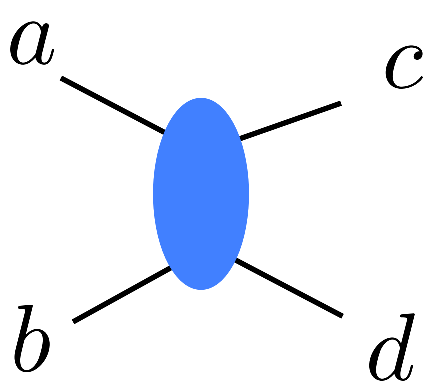
- same structure for 1PI:

$$\tilde{\Omega}_{ab\to cX}\left(N,\hat{\eta},\alpha_s(\mu),\frac{\mu^2}{s}\right) = \Delta_a^N\,\Delta_b^N\,\Delta_c^{N+1}\,J_d^N\left(\alpha_s(\mu),\frac{\mu^2}{\hat{s}}\right)$$

$$\times\ \mathrm{Tr}\,\{\,H\,S\,\}_{ab\to cd}$$

$$J_d^N = \exp\left\{\int_0^1 dz\frac{z^N-1}{1-z}\left[\int_{(1-z)^2\hat{s}}^{(1-z)\hat{s}}\frac{dq^2}{q^2}A_d(\alpha_s(q^2))+\frac{1}{2}B_d(\alpha_s((1-z)\hat{s}))\right]\right\}$$

Compare leading logarithms ($\overline{\text{MS}}$):



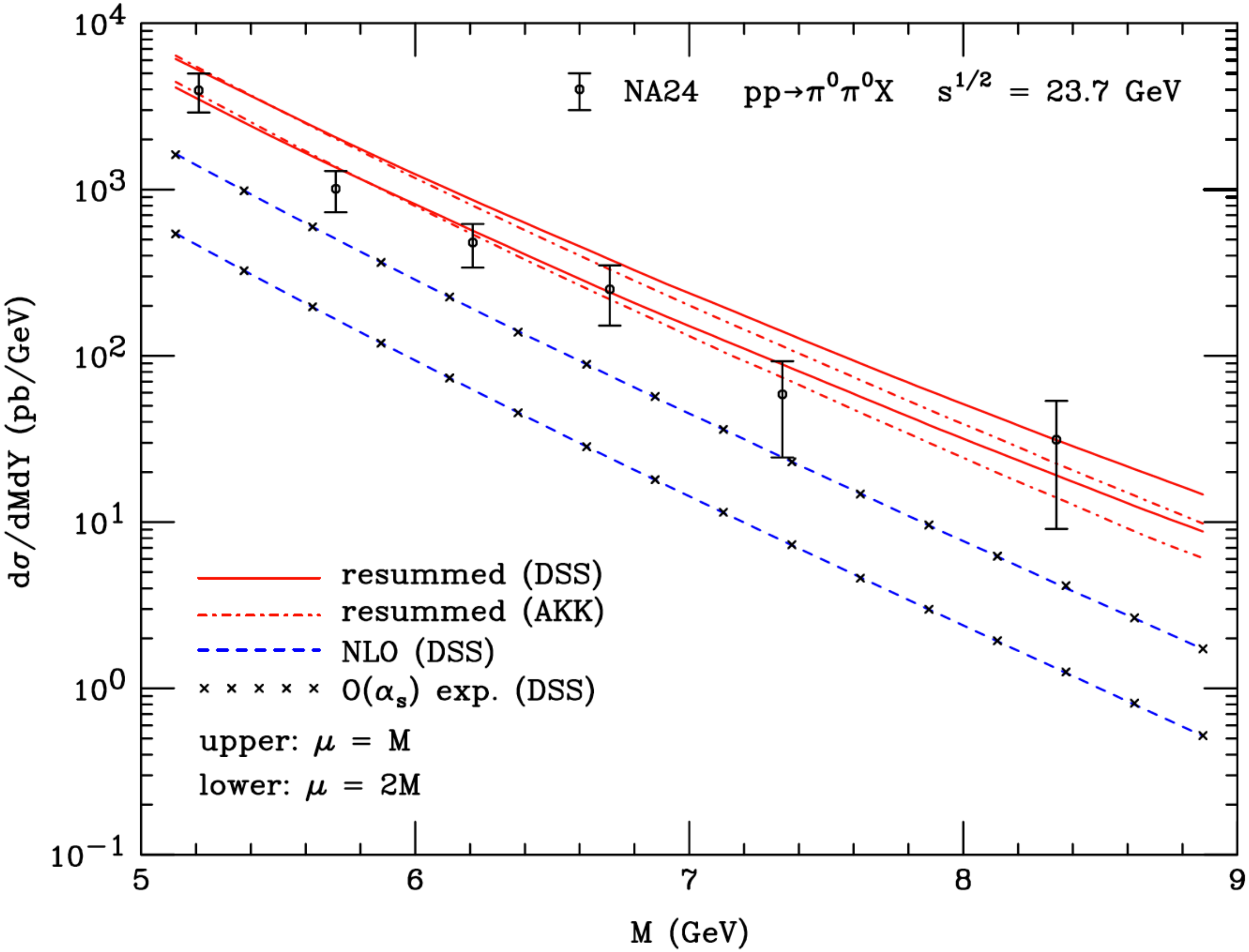
PIM: $$\sim \exp\left[\left(C_a + C_b + C_c + C_d\right) \frac{\alpha_s}{\pi} \ln^2 N\right]$$

1PI: $$\sim \exp\left[\left(C_a + C_b + C_c - \frac{1}{2} C_d\right) \frac{\alpha_s}{\pi} \ln^2 N\right]$$

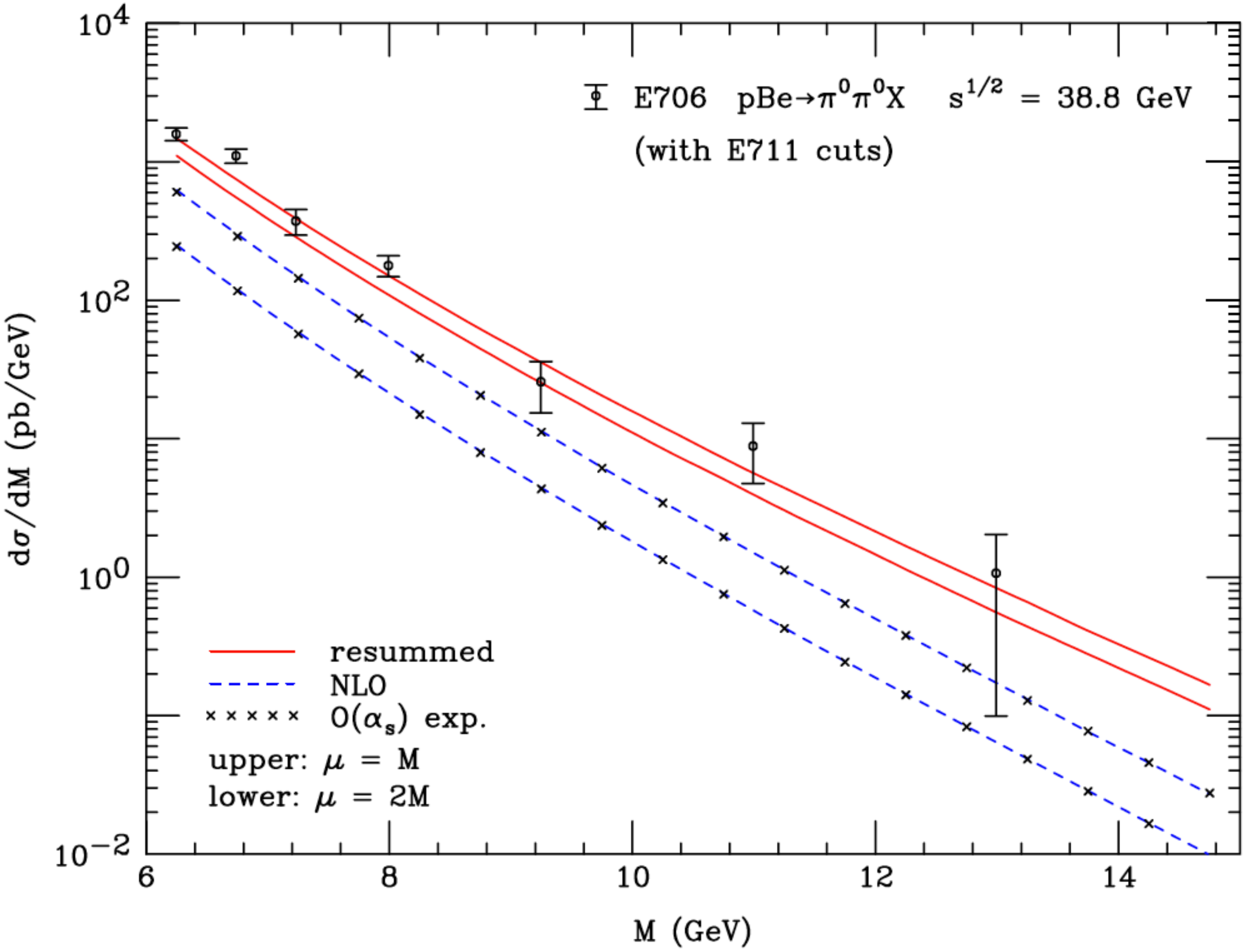
$$(C_q = C_F ,\ C_g = C_A)$$

Resummation for pp → h_1 h_2 X

L. Almeida, G.Sterman, WV



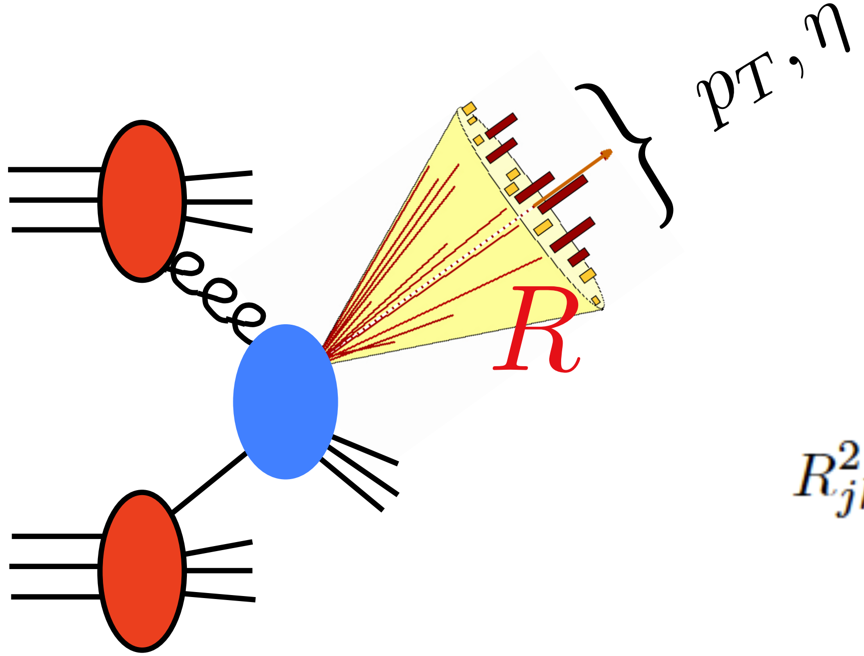
NA24 pp→π⁰π⁰X s^1/2 = 23.7 GeV
resummed (DSS)
resummed (AKK)
NLO (DSS)
O(αs) exp. (DSS)
upper: μ = M
lower: μ = 2M
dσ/dMdY (pb/GeV)
M (GeV)



E706 pBe→π⁰π⁰X s^{1/2} = 38.8 GeV
(with E711 cuts)
resummed
NLO
O(α_s) exp.
upper: μ = M
lower: μ = 2M
dσ/dM (pb/GeV)
M (GeV)
10^4
10^2
10^0
10^-2
6
8
10
12
14

Resummation for pp → jet X

D.de Florian, P.Hinderer, A.Mukherjee, F.Ringer, WV
(PRL 2014)



$$d_{jk} \equiv \min(k_{T_j}^{2p}, k_{T_k}^{2p}) \frac{R_{jk}^2}{R^2}$$

$$R_{jk}^2 \equiv (\eta_j - \eta_k)^2 + (\phi_j - \phi_k)^2$$

- recall, 1PI:

$$\tilde{\Omega}_{ab\to cX}\left(N, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{s}\right) = \Delta_a^N \, \Delta_b^N \, \Delta_c^{N+1} J_d^N$$

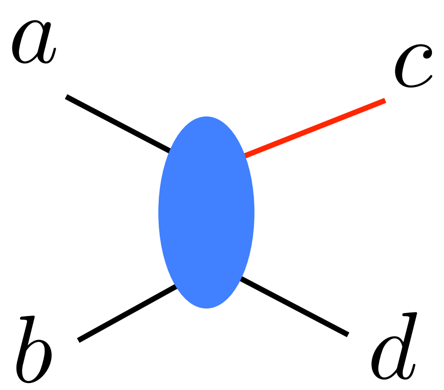
?

$$\times \, \mathrm{Tr}\,\{ H \, S \}_{ab\to cd}$$

Threshold logarithms depend crucially on treatment of jet:

Kidonakis, Sterman

LO:



$\Rightarrow$ jet massless

(1) keep jet massless at threshold:

$$\sim \ \exp\left[\left(C_a + C_b - \frac{1}{2}C_c - \frac{1}{2}C_d\right)\frac{\alpha_s}{\pi}\ln^2 N\right]$$

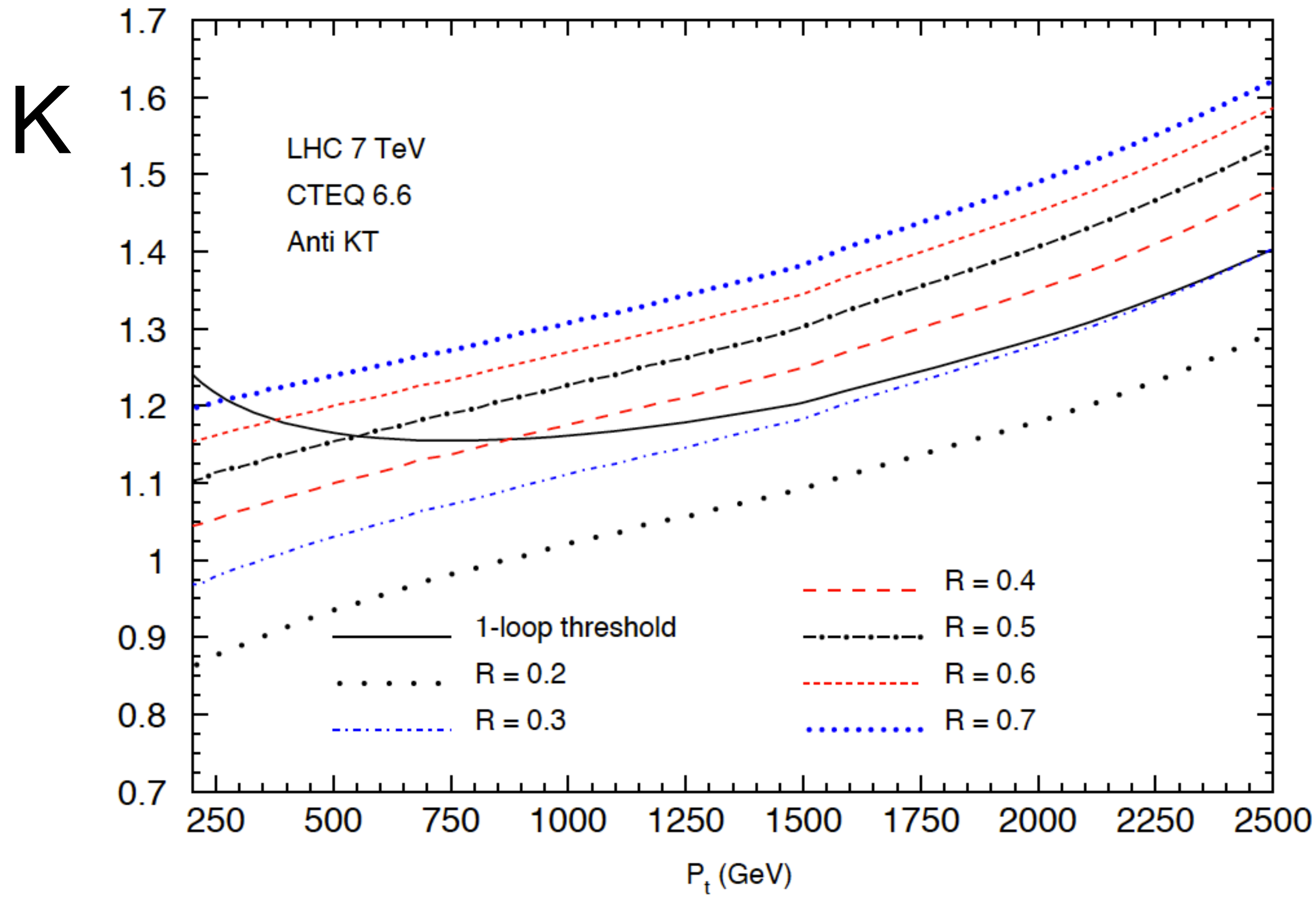
no dependence on R

Kidonakis, Owens;
Moch, Kumar

(2) jet allowed to be massive at threshold:

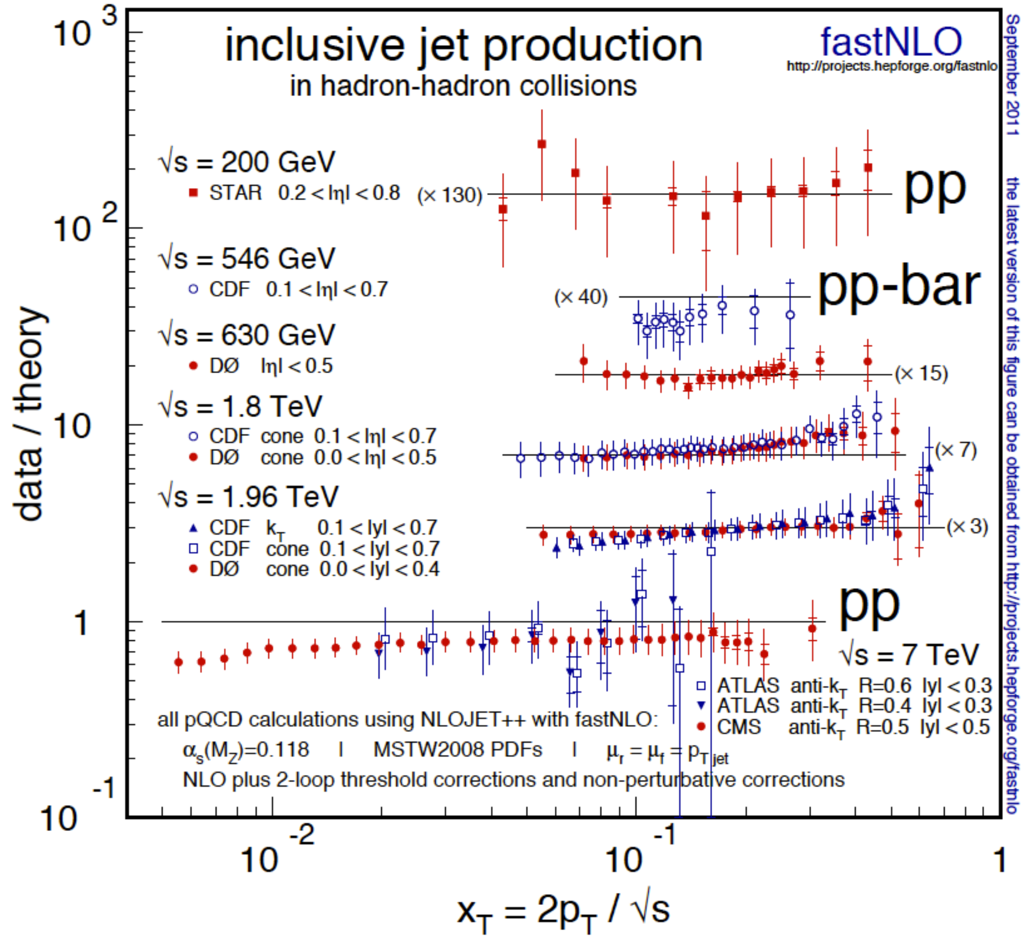
$$\sim \ \exp\left[\left(C_a + C_b - \frac{1}{2}C_d\right)\frac{\alpha_s}{\pi}\ln^2 N + \frac{\alpha_s}{\pi}C_c \ln(R)\ln(N)\right]$$

LHC



Moch, Kumar (arXiv:1309.5311)
based on (1)

Wobisch et al.



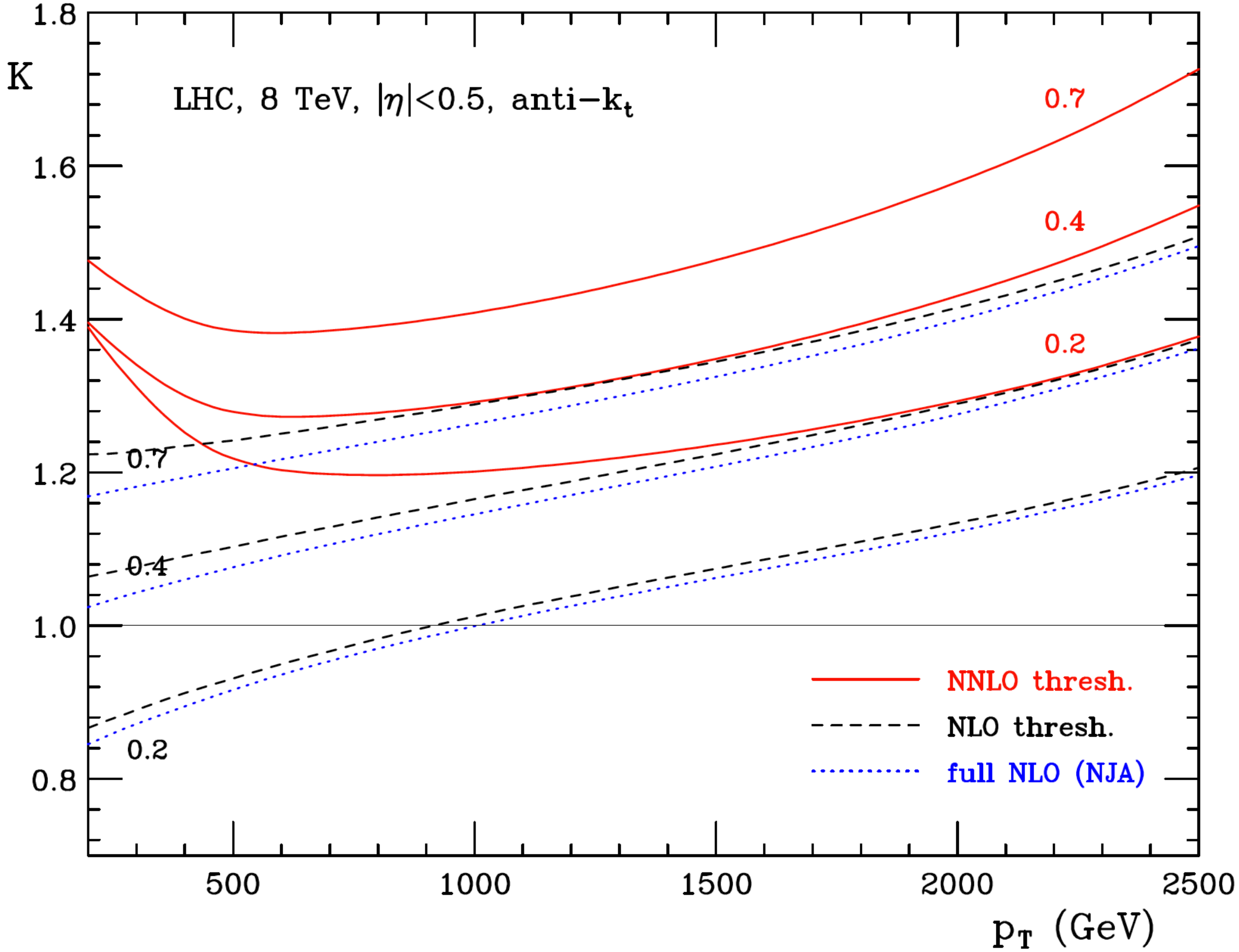
- More recent study of jet-pdf interplay: Watt, Motylinski, Thorne

Full (analytical) NLO calculation for “narrow jets”

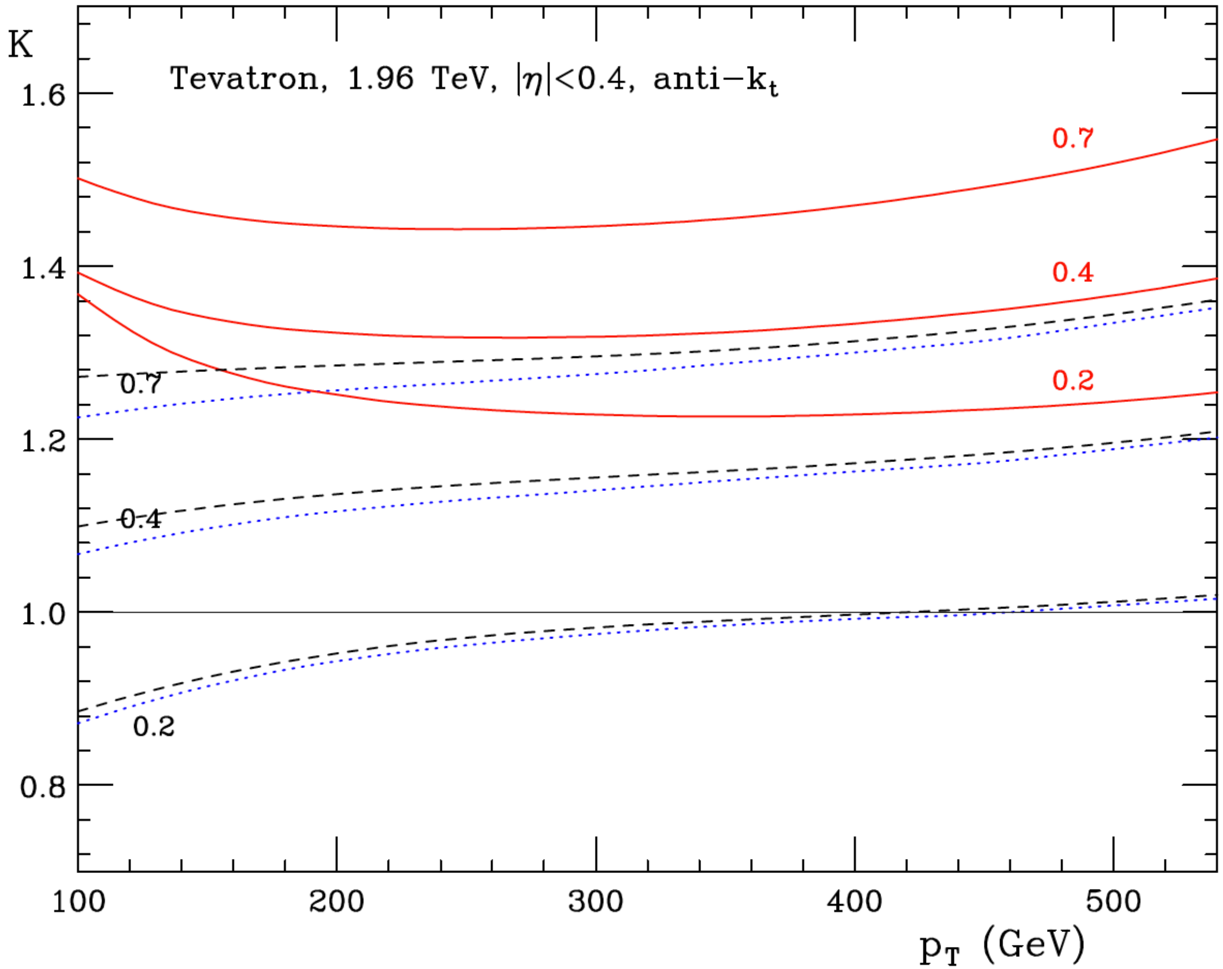
Jäger, Stratmann, WV; Mukherjee, WV

- accurate to better than 2-3%
- allows to pin down behavior near threshold:
  → confirms that (2) is right

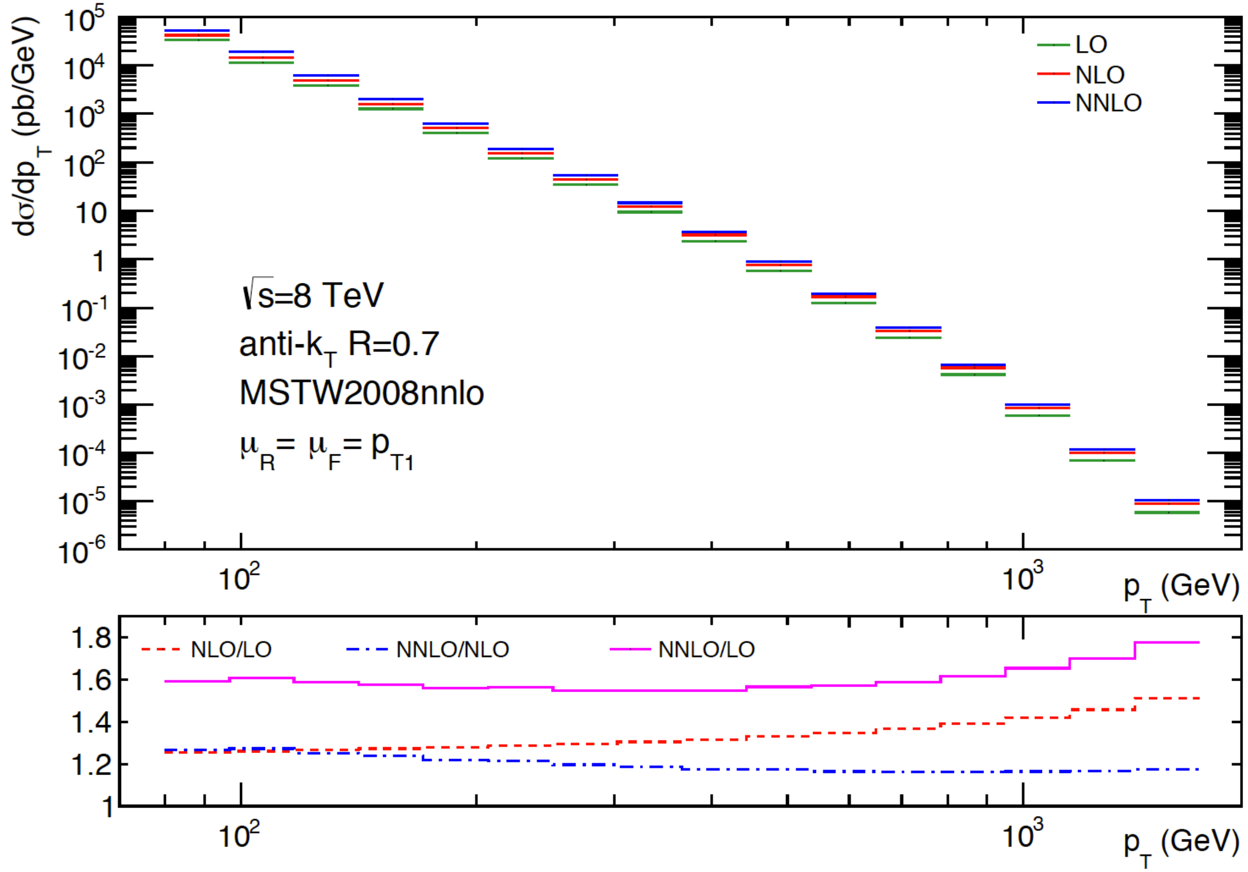
de Florian, WV;
de Florian, Hinderer,
Mukherjee, Ringer, WV



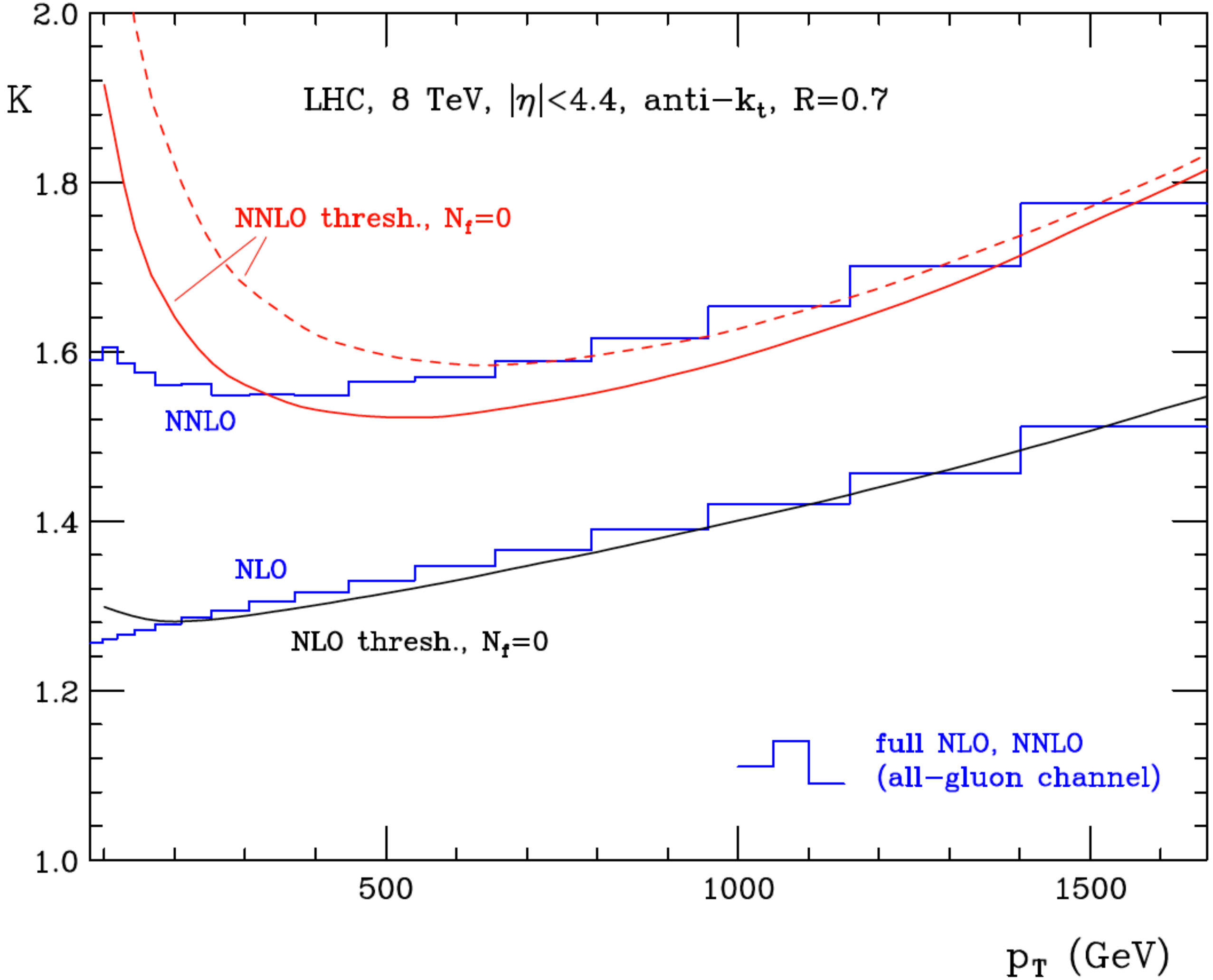
K
LHC, 8 TeV, |η|<0.5, anti-k_t
0.7
0.4
0.2
0.7
0.4
0.2
NNLO thresh.
NLO thresh.
full NLO (NJA)
1.8
1.6
1.4
1.2
1.0
0.8
500
1000
1500
2000
2500
p_T (GeV)


K
Tevatron, 1.96 TeV, |η|<0.4, anti-k_t
0.7
0.4
0.2
0.7
0.4
0.2
1.6
1.4
1.2
1.0
0.8
100
200
300
400
500
p_T (GeV)

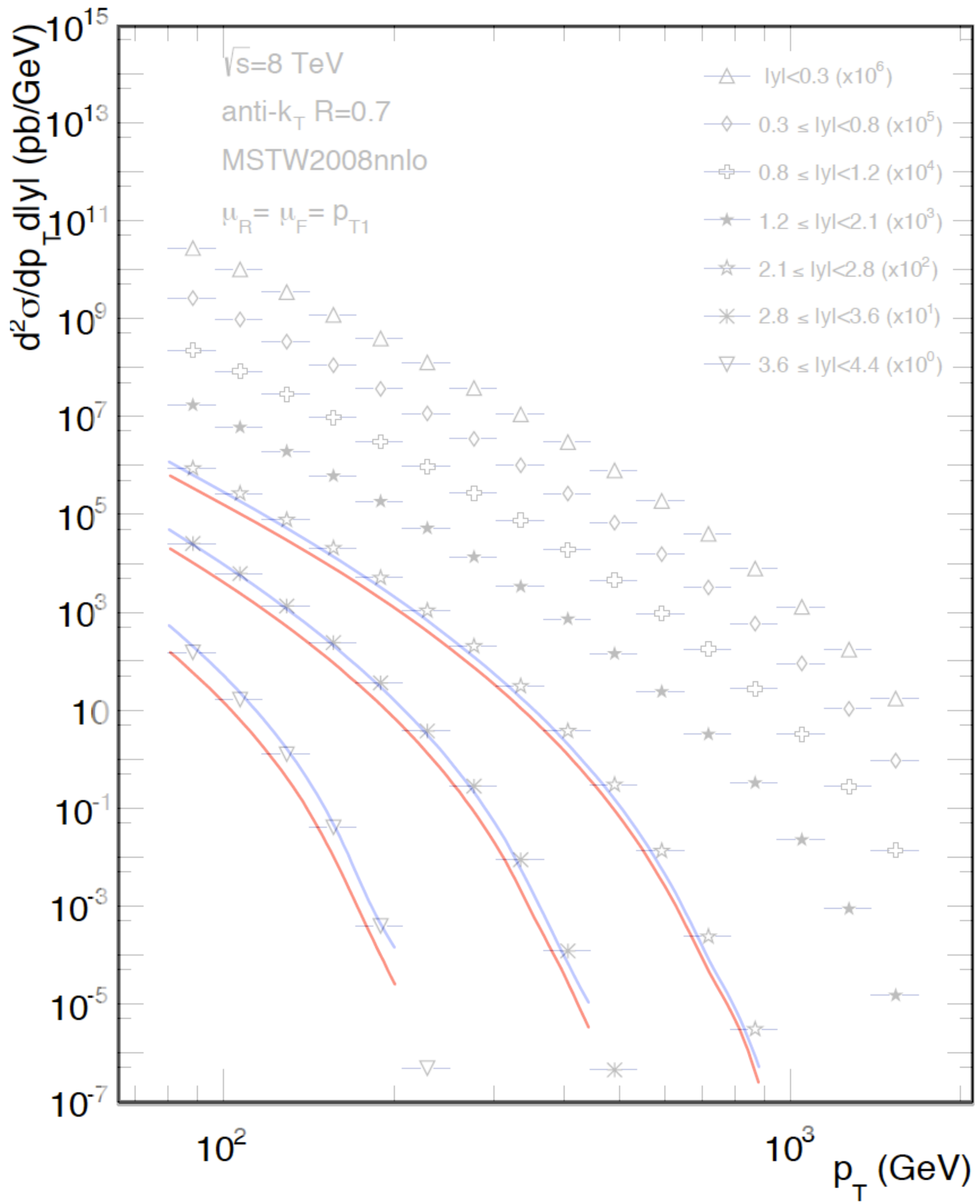
- NNLO corrections in all-gluon channel:



Currie, Gehrmann-De Ridder, Glover, Pires, arXiv:1310.3993



LHC, 8 TeV, |η|<4.4, anti−k$_t$, R=0.7
K
NNLO thresh., N$_f$=0
NNLO
NLO
NLO thresh., N$_f$=0
full NLO, NNLO
(all−gluon channel)
p$_T$ (GeV)
2.0
1.8
1.6
1.4
1.2
1.0
500
1000
1500



NNLO

NLO

Currie et al.
arXiv:1310.3993
“gg only”

Conclusions:

- significant resummation effects in many hadronic scattering processes
- improve theoretical framework, relevant for phenomenology
- predictions from resummation formalism serve as benchmark for full NNLO calculations