LHC v SUSY

by

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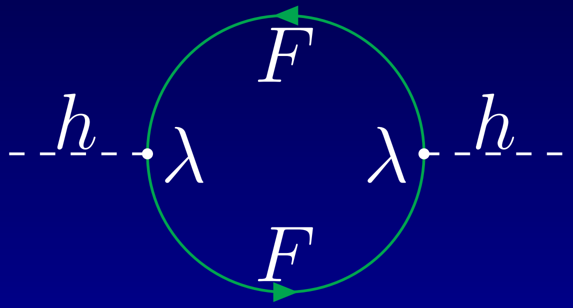
Talk outline

- Weak scale SUSY
- Fits and 2011 LHC searches
- mAMSB interpretation
- 2012 searches

Please ask questions while I'm talking

Technical Hierarchy Problem

A problem with light, fundamental scalars. Their mass receives quantum corrections from heavy particles in the theory:



$$\sim -\frac{c\lambda^2}{16\pi^2}\int \frac{d^n k}{k^2 - m_F^2} + \ldots$$

Quantum correction to Higgs mass:

$$m_h^{phys} = m_h^{tree} + \mathcal{O}(m_F/100).$$

$m_F \sim 10^{19}$ GeV$/c^2$ is *heaviest mass scale* present. Higgs is eaten by W, Z to give $O(M_{W,Z}) \sim 90$ GeV/c^2 $\Rightarrow m_h^{tot} \lesssim 1$ TeV$/c^2$.

Symmetry

Standard model gauge symmetry is *internal*, but supersymmetry (SUSY) is a space-time symmetry. We call extra SUSY generators $Q, \bar{Q}$.

$$\begin{aligned} Q|\text{fermion}\rangle &\rightarrow |\text{boson}\rangle \\ Q|\text{boson}\rangle &\rightarrow |\text{fermion}\rangle \end{aligned}$$

In the simplest form of SUSY, we have multiplets

$$\begin{pmatrix} \text{spin } 0 \\ \text{spin } 1/2 \end{pmatrix}, \qquad \begin{pmatrix} \text{spin } 1/2 \\ \text{spin } 1 \end{pmatrix},$$

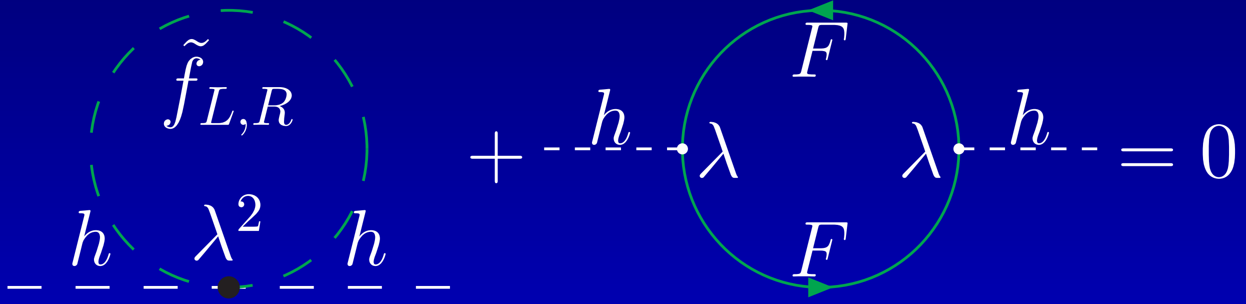
where each spin component in the multiplet should have identical quantum numbers (except spin).

Supersymmetric Solution

Exact supersymmetry adds 2 scalars $\tilde{f}_{L,R}$ for every massive fermion with

$$m_{\tilde{f}_{L,R}} = m_F$$

and they couple to h with the same strength:



Q: Where are the selectrons?

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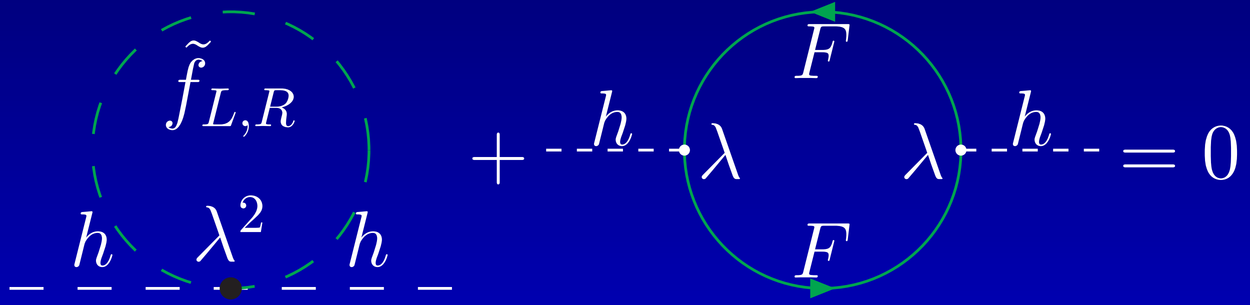

$\mathcal{Q}$: Where are the selectrons?
$\mathcal{A}$: SUSY must be softly broken.

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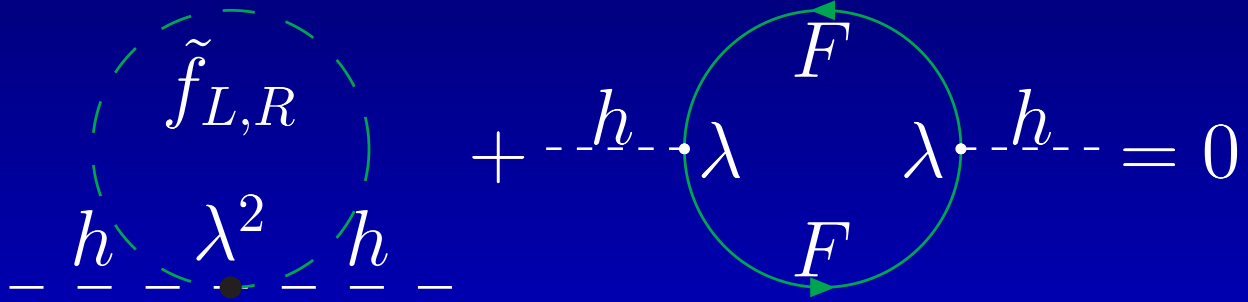

When we break SUSY, we must make sure that we don't reintroduce the naturalness problem: "soft breaking".

Soft breaking

Make scalar partners heavier than fermions:

$$m^2_{\tilde{f}_{L,R}} = m^2_F + \delta^2$$

Then we find a quantum correction to m_h of (Drees)

$$\Delta m_h^2 \sim \frac{\lambda^2}{16\pi^2} \left(4\delta^2 + 2\delta^2 \ln \frac{m_F^2}{\mu^2} \right) + O(\delta^4).$$

So, if $\delta \lesssim O(1)$ TeV/c^2, there's no fine tuning in m_h.

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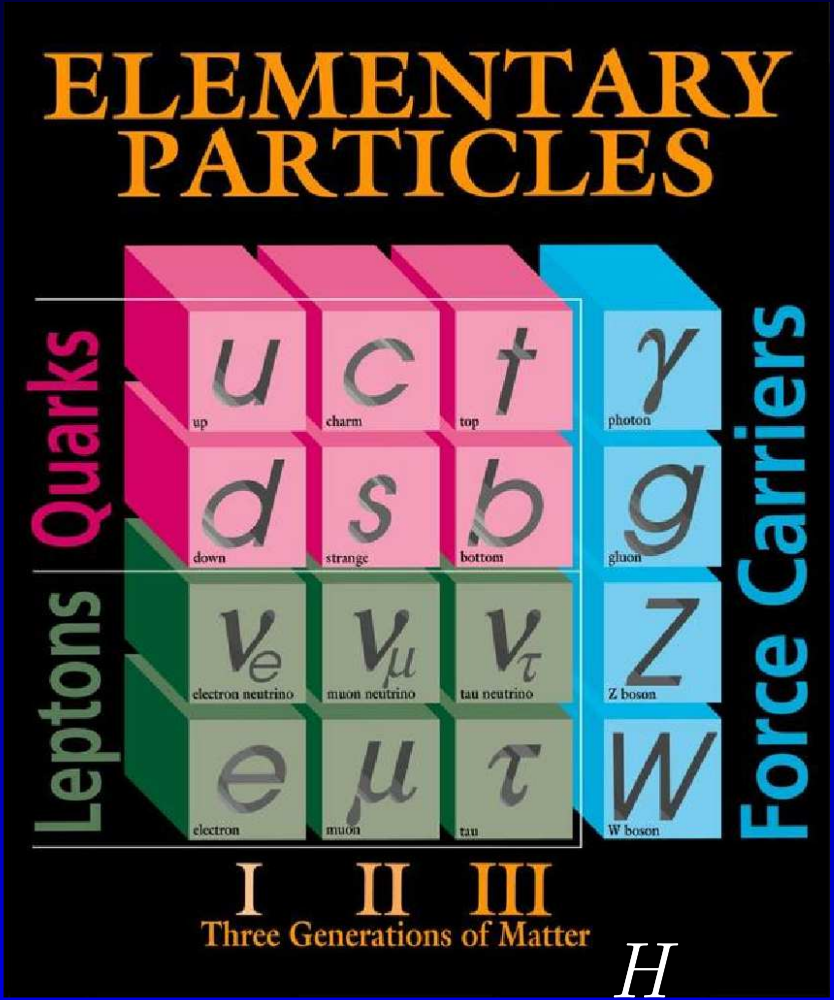
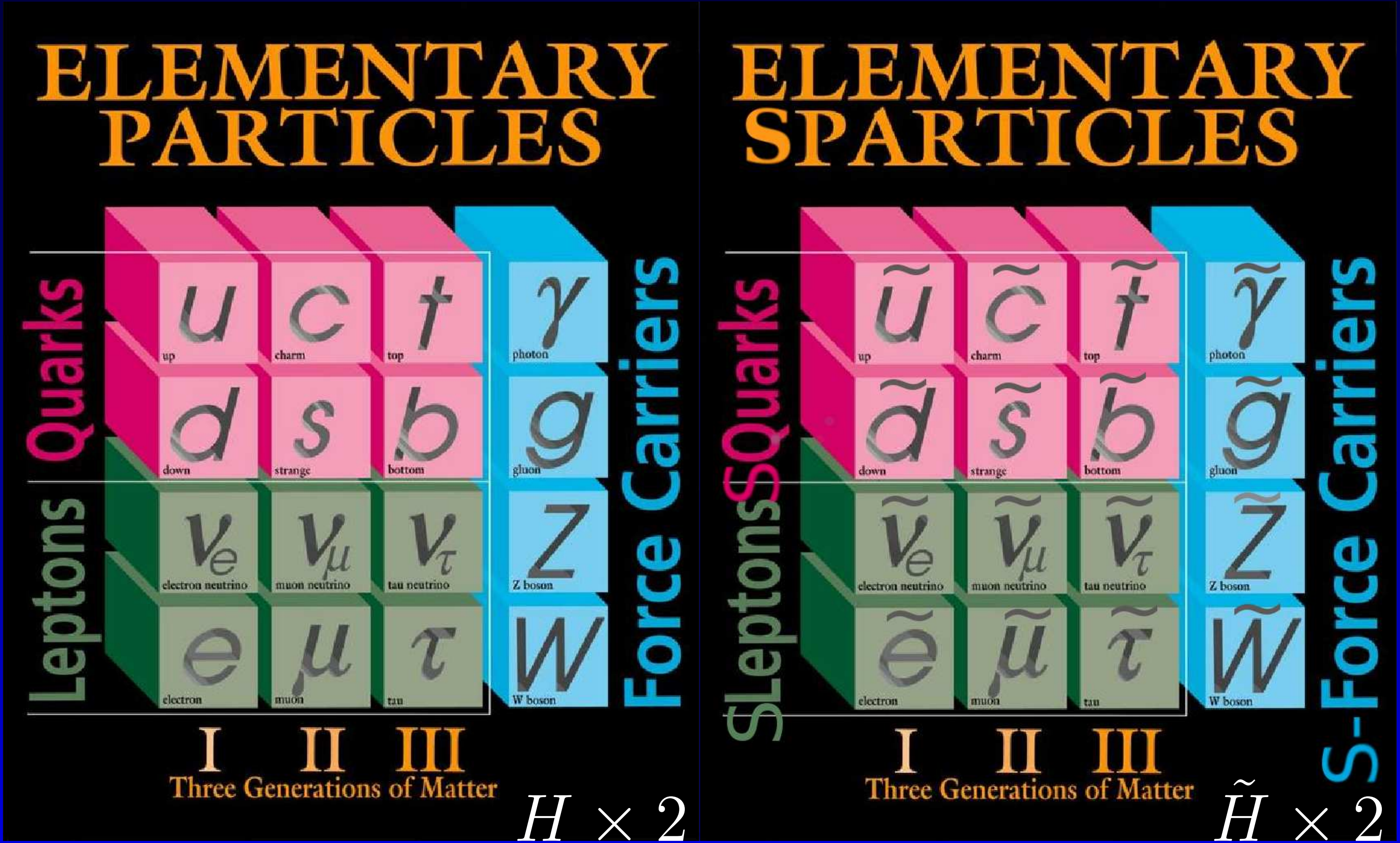
So, if $\delta \stackrel{<}{\sim} O(1)$ TeV/c^2, there's no fine tuning in m_h. *We should see supersymmetric particles in the Large Hadron Collider.*

Supersymmetric Copies



H

Supersymmetric Copies



$H \times 2$ $\tilde{H} \times 2$

Broken Symmetry

3 components of the Higgs particles are eaten by $W^\pm$, Z^0, leaving us with 5 physical states:

$$h^0, H^0\text{(CP+)}, \qquad A^0\text{(CP-)}, \qquad H^\pm$$

SUSY breaking and electroweak breaking imply particles with identical quantum numbers mix:

$$\begin{aligned}
(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0) &\rightarrow \chi^0_{1,2,3,4} \\
(\tilde{t}_L, \tilde{t}_R) &\rightarrow \tilde{t}_{1,2} \\
(\tilde{b}_L, \tilde{b}_R) &\rightarrow \tilde{b}_{1,2} \\
(\tilde{\tau}_L, \tilde{\tau}_R) &\rightarrow \tilde{\tau}_{1,2} \\
(\tilde{W}^\pm, \tilde{H}^\pm) &\rightarrow \chi^\pm_{1,2}
\end{aligned}$$

Universality

Reduces number of SUSY breaking parameters from 100 to 3:

- $\tan\beta \equiv v_2/v_1$
- m_0, the common scalar mass (flavour).
- $M_{1/2}$, the common gaugino mass (GUT/string).
- A_0, the common trilinear coupling (flavour).

These conditions should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

$$\propto \ 1/(16\pi^2)\ln(M_X/M_Z).$$

Also, Higgs potential parameter sgn(μ)=±1.

Implementation

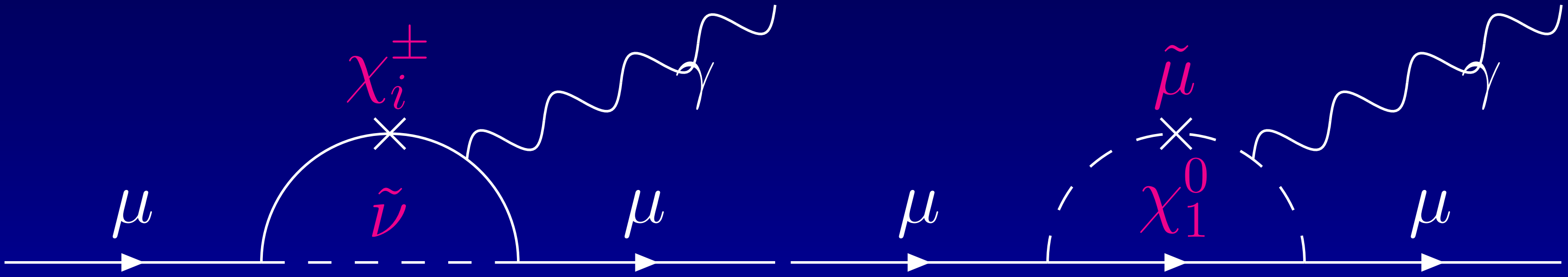
We use

- *95% C.L. direct search constraints*
- $\Omega_{DM}h^2 = 0.1143 \pm 0.02$ `micrOMEGAs`
- $\delta(g-2)_\mu/2 = (29.5 \pm 8.8) \times 10^{-10}$ Stöckinger *et al*
- $B-$physics observables including `SusyBSG` $BR[b \to s\gamma]_{E_\gamma > 1.6 \text{ GeV}} = (3.52 \pm 0.38) \times 10^{-4}$, $\text{BR}(B_s \to \mu\mu) < 1.1 \times 10^{-8}$ `micrOMEGAs`
- Electroweak data W Hollik, A Weber *et al*

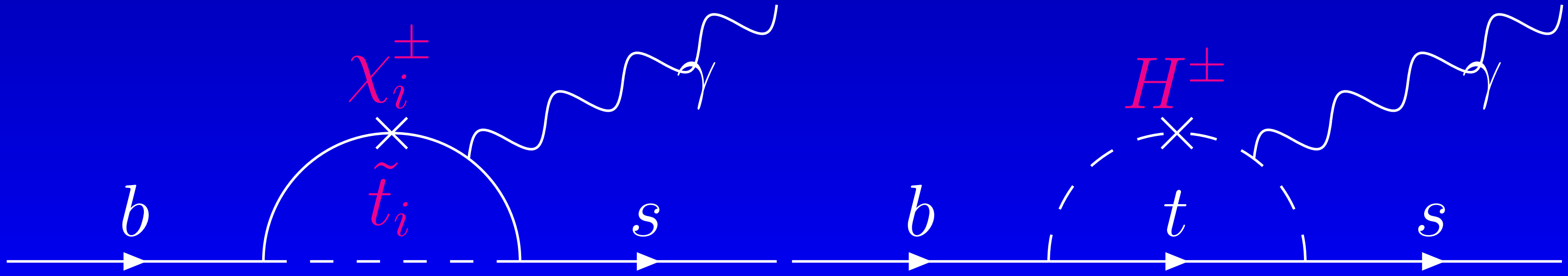
$$2 \ln \mathcal{L} = -\sum_i \chi_i^2 + c = \sum_i \frac{(p_i - e_i)^2}{\sigma_i^2} + c$$

Additional observables

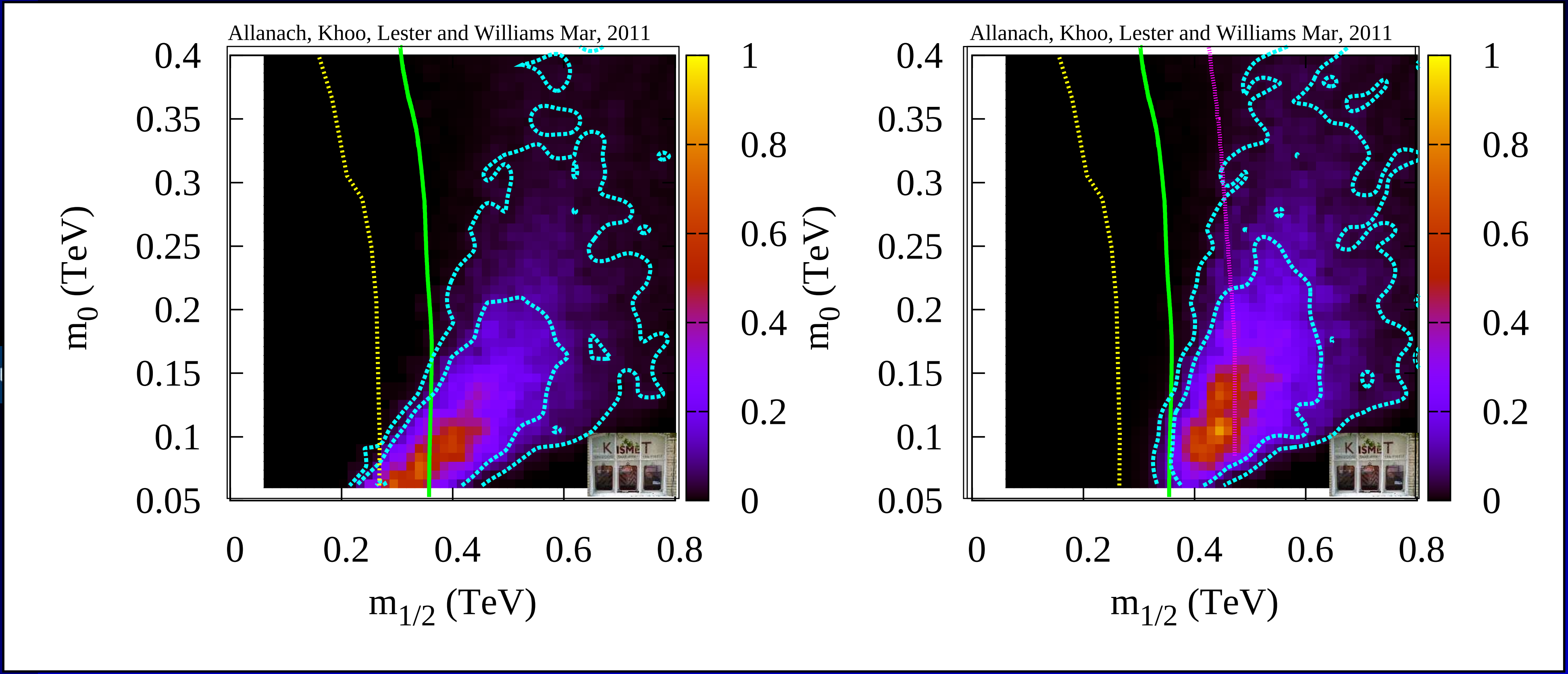
$$\delta\frac{(g-2)_\mu}{2} \sim 13 \times 10^{-10} \left(\frac{100\text{ GeV}}{M_{SUSY}}\right)^2 \tan\beta$$



$$BR[b \to s\gamma] \propto \tan\beta(M_W/M_{SUSY})^2$$

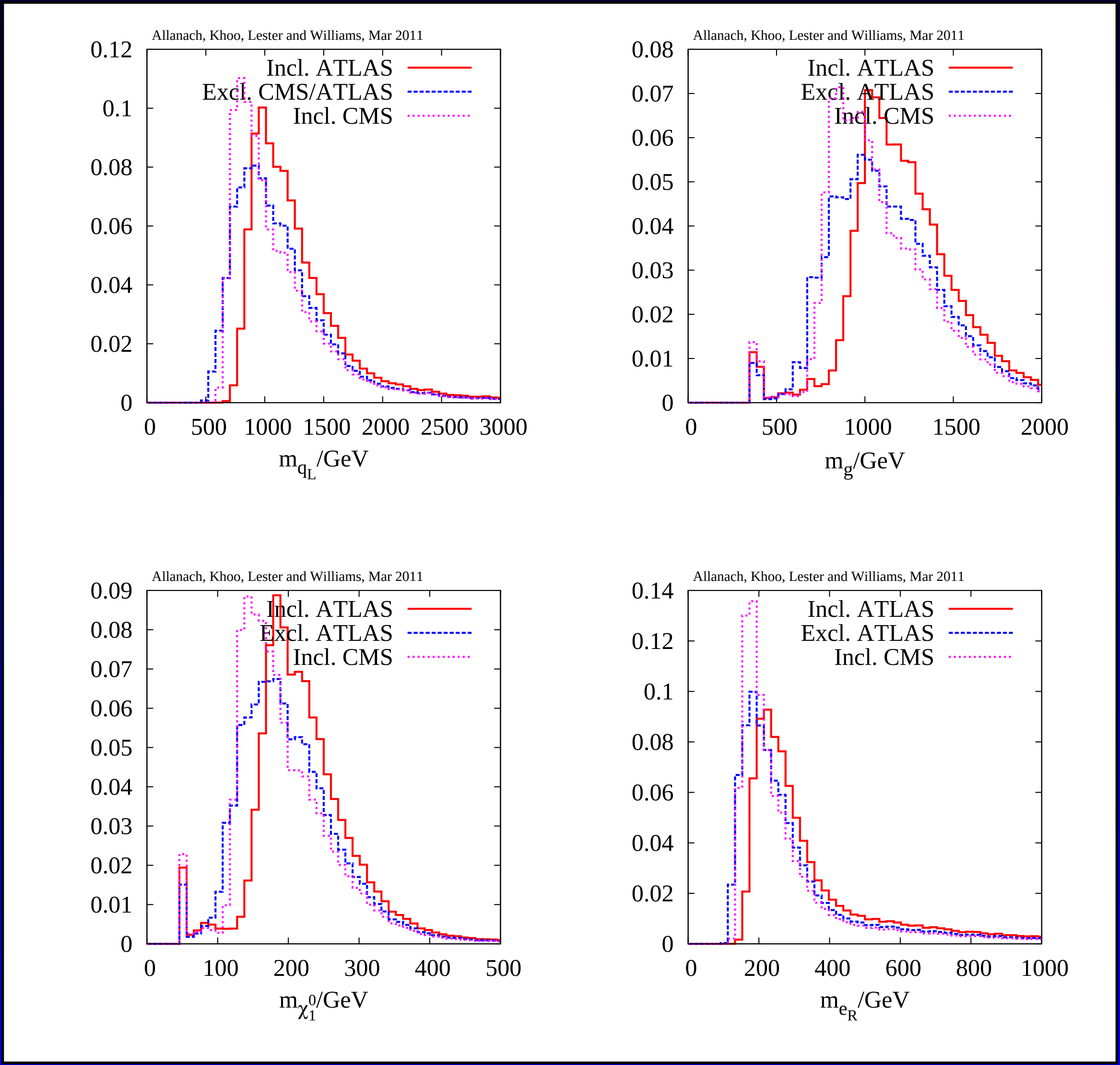


ATLAS Weighted Fits



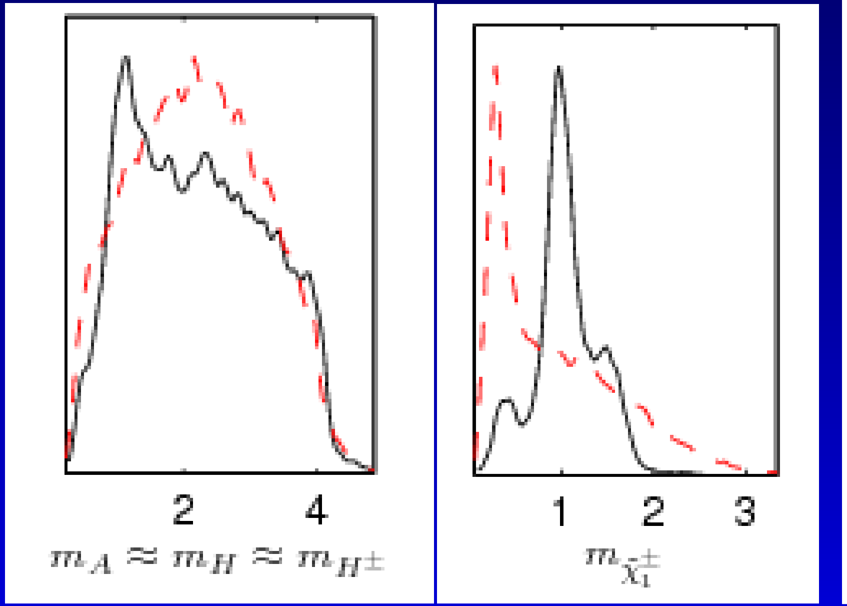
Again, we assume A_0-$\tan\beta$ independence and interpolate across m_0 and $m_{1/2}$. CMS 35 pb^{-1}, ATLAS 35 pb^{-1}, CMS 1 fb^{-1}

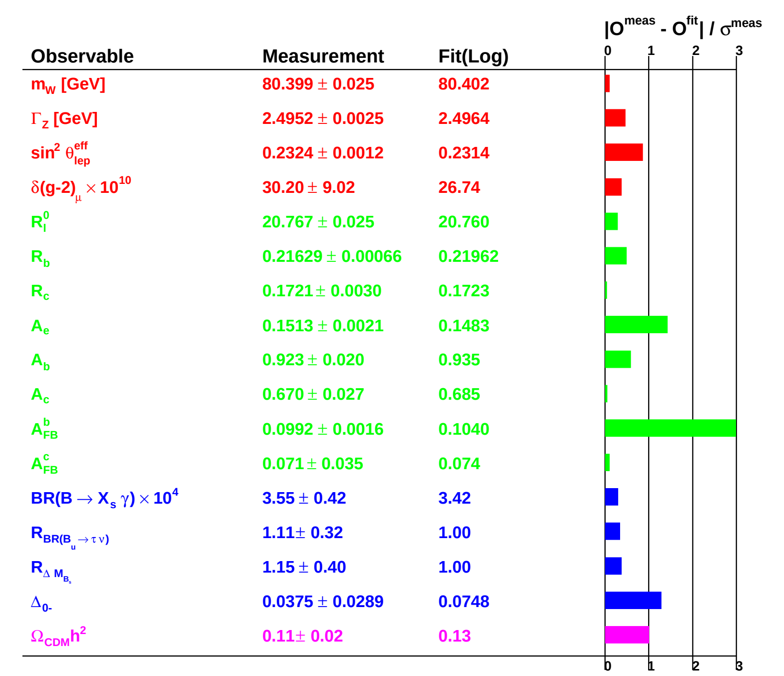
CMS/ATLAS Weighted Fits



pMSSM Fits

25 pMSSM input parameters are: $M_{1,2,3}$, $A_{t,b,\tau,\mu}$, $m_{H_{1,2}}$, $\tan\beta$, $m_{\tilde{d}_{R,L}} = m_{\tilde{s}_{R,L}}$, $m_{\tilde{u}_{R,L}} = m_{\tilde{c}_{R,L}}$, $m_{\tilde{e}_{R,L}} = m_{\tilde{\mu}_{R,L}}$, $m_{\tilde{t},\tilde{b},\tilde{\tau}_{R,L}}$ m_t, $m_b(m_b)$ $\alpha_s(M_Z)^{\overline{MS}}$, $\alpha^{-1}(M_Z)^{\overline{MS}}$, M_Z. Combined Bayesian fit[a]:





Observable	Measurement	Fit(Log)
m_W [GeV]	80.399 ± 0.025	80.402
Γ_Z [GeV]	2.4952 ± 0.0025	2.4964
$\sin^2\theta^{eff}_{lep}$	0.2324 ± 0.0012	0.2314
$\delta(g\text{-}2)_\mu \times 10^{10}$	30.20 ± 9.02	26.74
R^0_l	20.767 ± 0.025	20.760
R_b	0.21629 ± 0.00066	0.21962
R_c	0.1721 ± 0.0030	0.1723
A_e	0.1513 ± 0.0021	0.1483
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.685
A^b_{FB}	0.0992 ± 0.0016	0.1040
A^c_{FB}	0.071 ± 0.035	0.074
$BR(B \to X_s\gamma) \times 10^4$	3.55 ± 0.42	3.42
$R_{BR(B_u \to \tau\nu)}$	1.11 ± 0.32	1.00
$R_{\Delta M_{B_s}}$	1.15 ± 0.40	1.00
Δ_{0-}	0.0375 ± 0.0289	0.0748
$\Omega_{CDM}h^2$	0.11 ± 0.02	0.13

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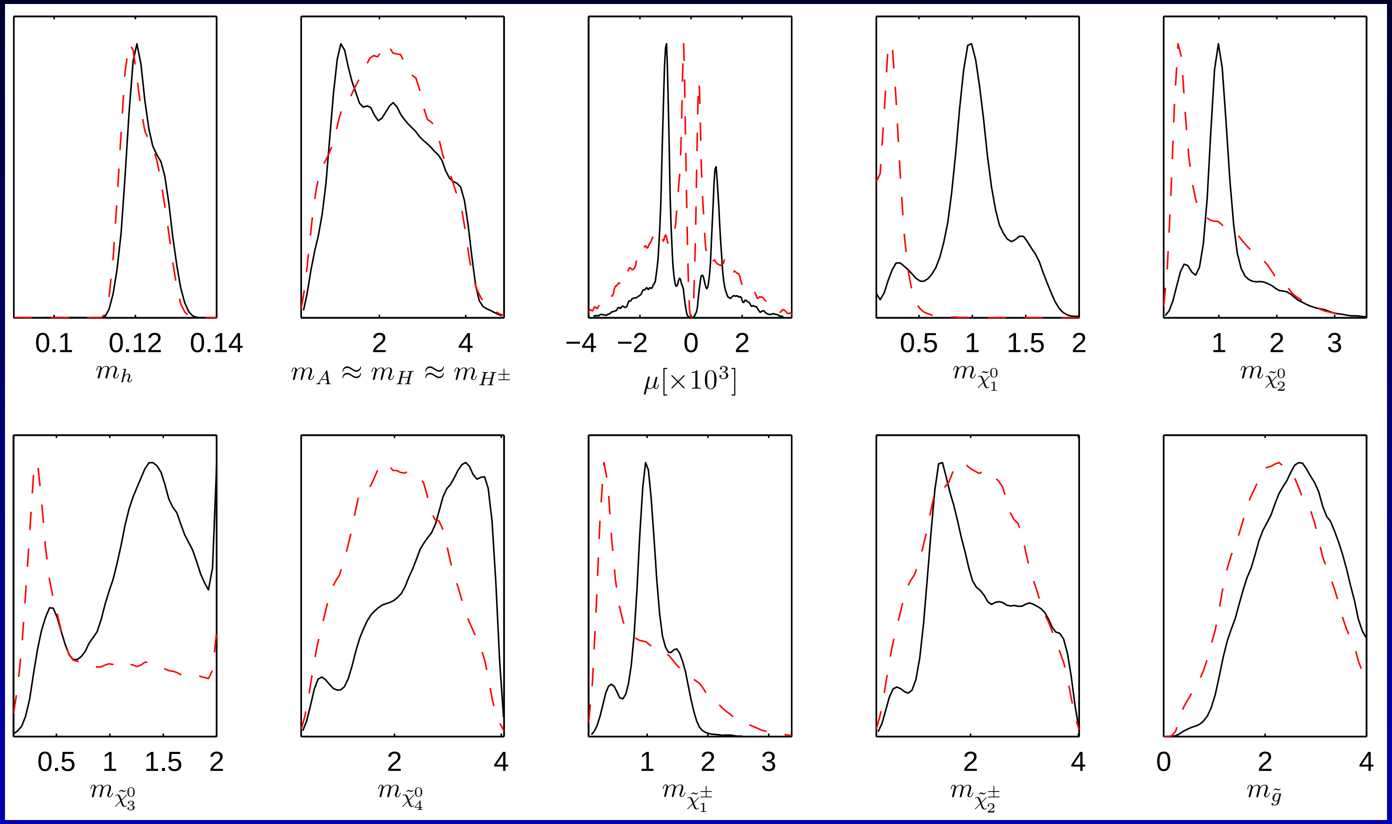
[a] S.S. AbdusSalam, BCA, F. Quevedo, F. Feroz, M. Hobson, PRD81 (2010) 985012, `arXiv:0904.2548`

Spectrum



Obtained with `MultiNest`[a] algorithm in 16 CPU years. Prior dependence is *useful*: which predictions are robust?

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[a] Feroz, Hobson `arxiv:0704.3704`

CMSSM at 1fb^{-1} is Getting Heavier

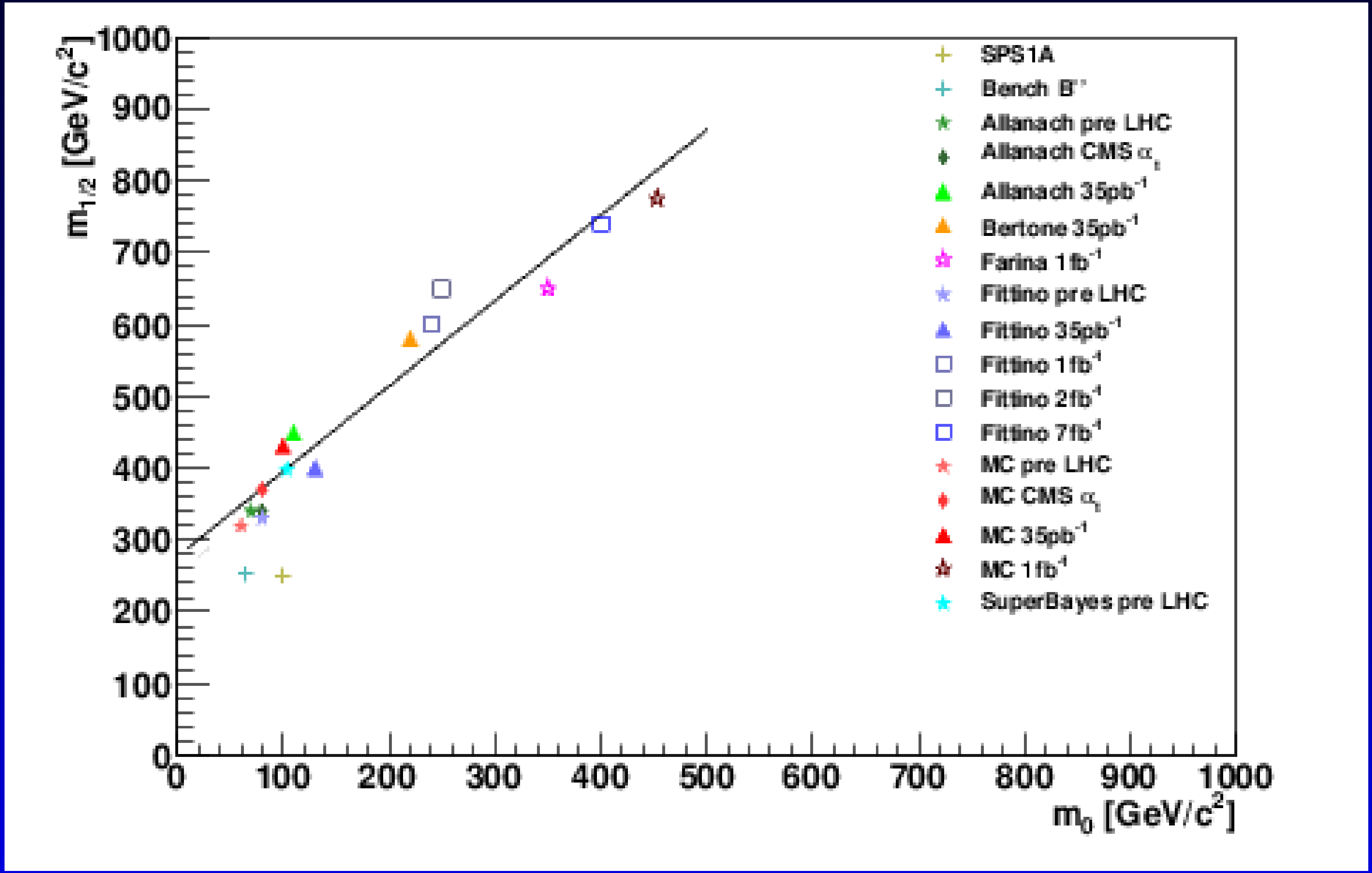
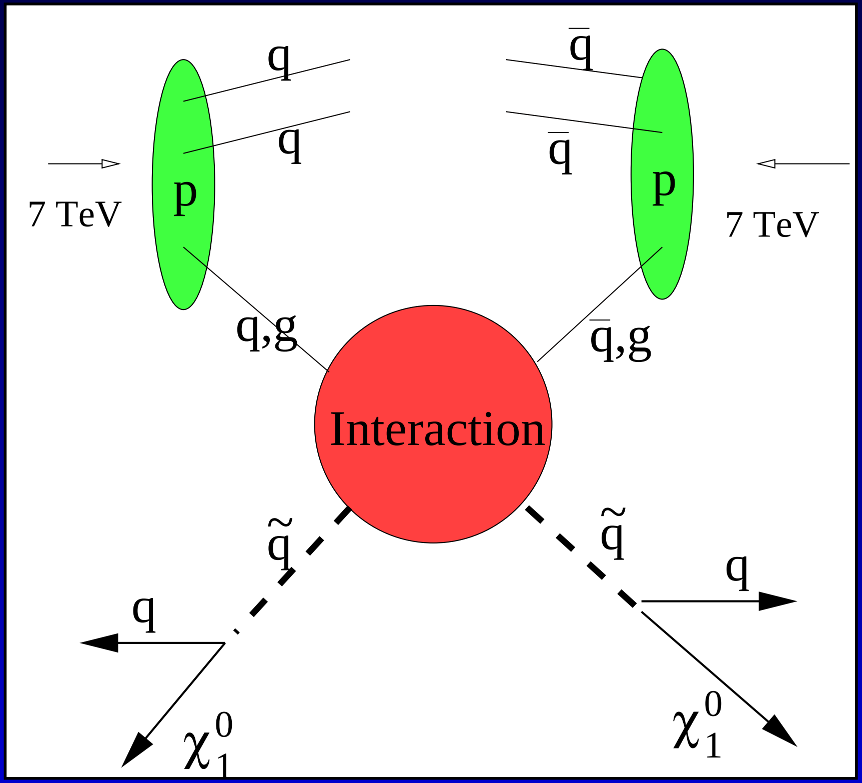


Figure 1: CMSSM good/best fit-points arXiv:1109.3859

Collider SUSY Dark Matter Production

Strong sparticle production and decay to dark matter particles.



Any (light enough) dark matter candidate that couples to hadrons can be produced at the LHC

Searches and the CMSSM

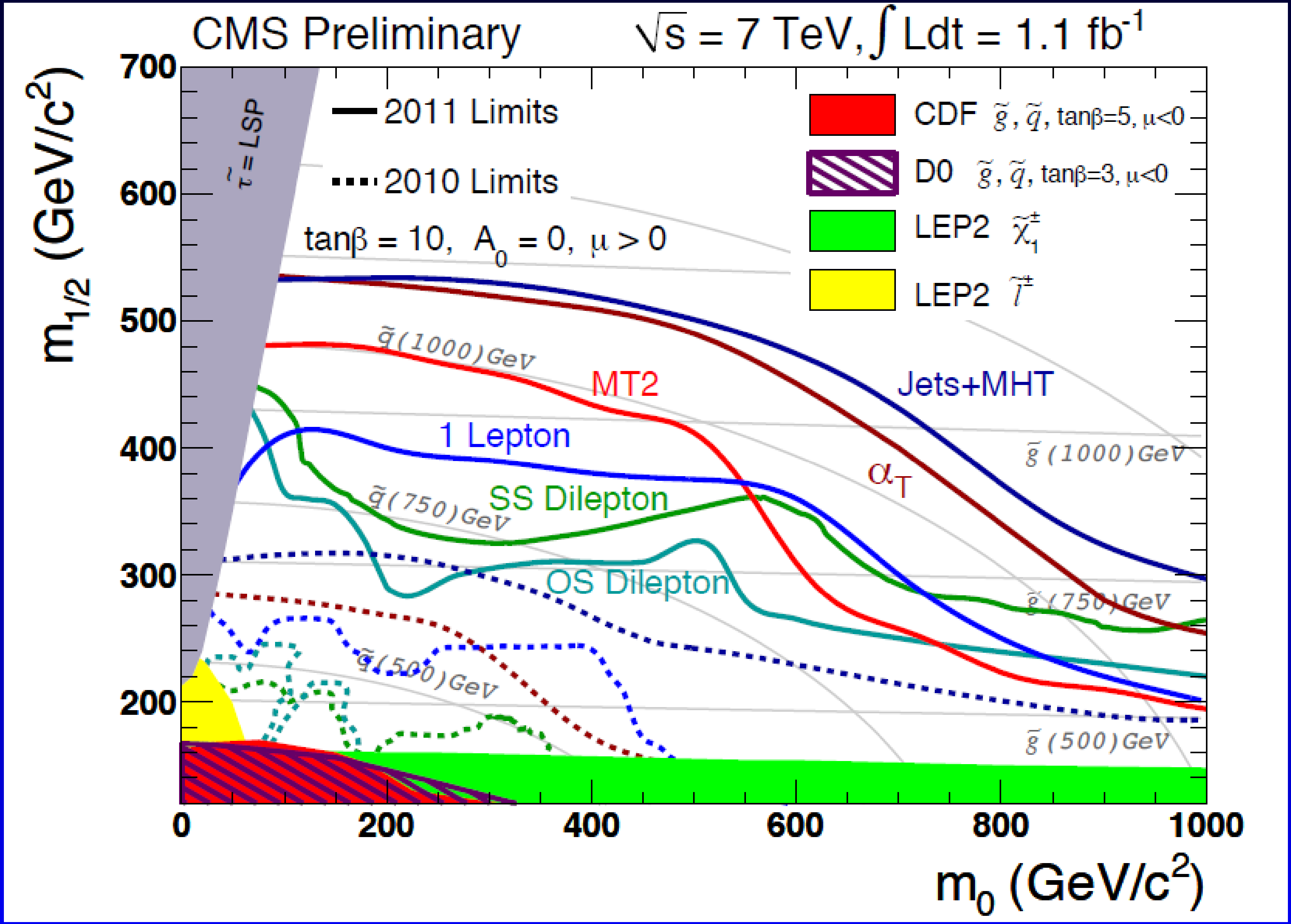
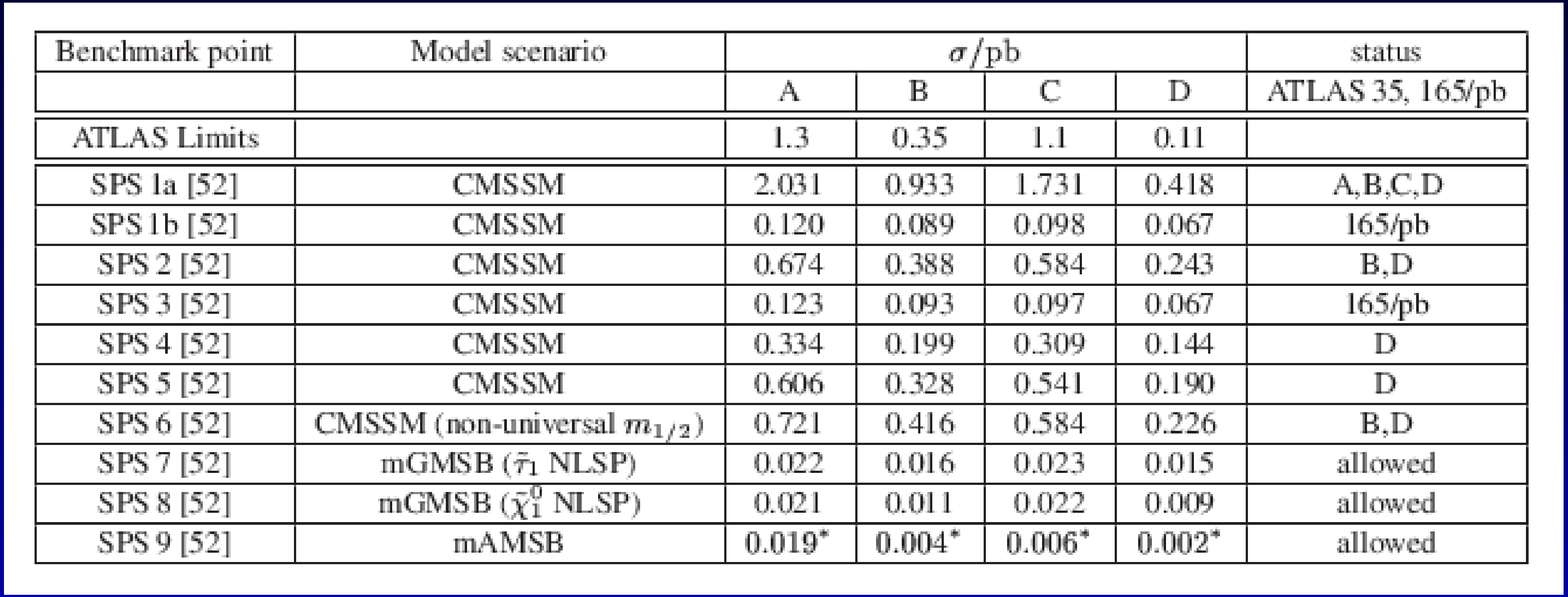


Figure 2: CMSSM CMS exclusions

SPS Points Savaged at 165 pb^{-1}



Benchmark point	Model scenario	$\sigma/$pb				status
		A	B	C	D	ATLAS 35, 165/pb
ATLAS Limits		1.3	0.35	1.1	0.11	
SPS 1a [52]	CMSSM	2.031	0.933	1.731	0.418	A,B,C,D
SPS 1b [52]	CMSSM	0.120	0.089	0.098	0.067	165/pb
SPS 2 [52]	CMSSM	0.674	0.388	0.584	0.243	B,D
SPS 3 [52]	CMSSM	0.123	0.093	0.097	0.067	165/pb
SPS 4 [52]	CMSSM	0.334	0.199	0.309	0.144	D
SPS 5 [52]	CMSSM	0.606	0.328	0.541	0.190	D
SPS 6 [52]	CMSSM (non-universal $m_{1/2}$)	0.721	0.416	0.584	0.226	B,D
SPS 7 [52]	mGMSB ($\tilde{\tau}_1$ NLSP)	0.022	0.016	0.023	0.015	allowed
SPS 8 [52]	mGMSB ($\tilde{\chi}_1^0$ NLSP)	0.021	0.011	0.022	0.009	allowed
SPS 9 [52]	mAMSB	0.019*	0.004*	0.006*	0.002*	allowed

Figure 3: Dolan, Grellscheid, Jaeckel, Khoze, Richardson, arXiv:1104.0585

Benchmarks

Currently we[a] have devised SUSY benchmark models. `1109.3859`

- CMSSM, NUHM, mAMSB, mGMSB, RPV and some simplified models (via pMSSM) are defined.
- Defining interesting *parameter planes*: identifying important parameters which control the masses of sparticles in each case.
- Discrete set of points along monotonic lines: next point for the experiments to study is defined as the lightest one that is not ruled out to 95% CL.

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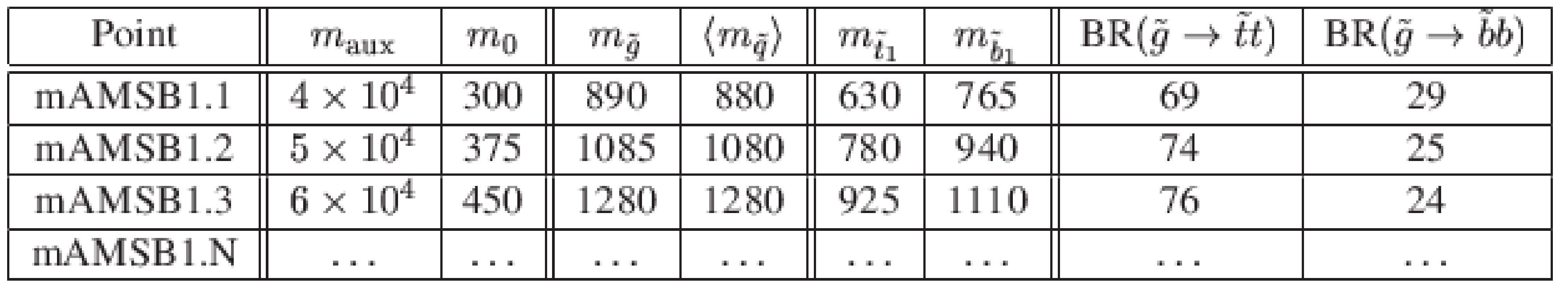
[a] S.S. AbdusSalam, BCA H. Dreiner, J. Ellis, S. Heinemeyer, M. Krämer, M. Mangano, K.A. Olive, S. Rogerson, L. Roszkowski, M. Schlaffer, G. Weiglein

Benchmarks Example: mAMSB

Plane: (m_{aux}, m_0) with $\tan\beta = 10, \mu > 0$.
Line: $m_0 = 0.0075 m_{\text{aux}}$, $\tan\beta = 10, \mu > 0$.
Points:



Point	m_{aux}	m_0	$m_{\tilde{g}}$	$\langle m_{\tilde{q}} \rangle$	$m_{\tilde{t}_1}$	$m_{\tilde{b}_1}$	BR$(\tilde{g} \to \tilde{t}t)$	BR$(\tilde{g} \to \tilde{b}b)$
mAMSB1.1	4×10^4	300	890	880	630	765	69	29
mAMSB1.2	5×10^4	375	1085	1080	780	940	74	25
mAMSB1.3	6×10^4	450	1280	1280	925	1110	76	24
mAMSB1.N	...	...	...	...	...	...	...	...

Figure 4: mAMSB benchmark points arXiv:1109.3859.

Next point for consideration is *the next one that hasn't been yet ruled out to 95%*

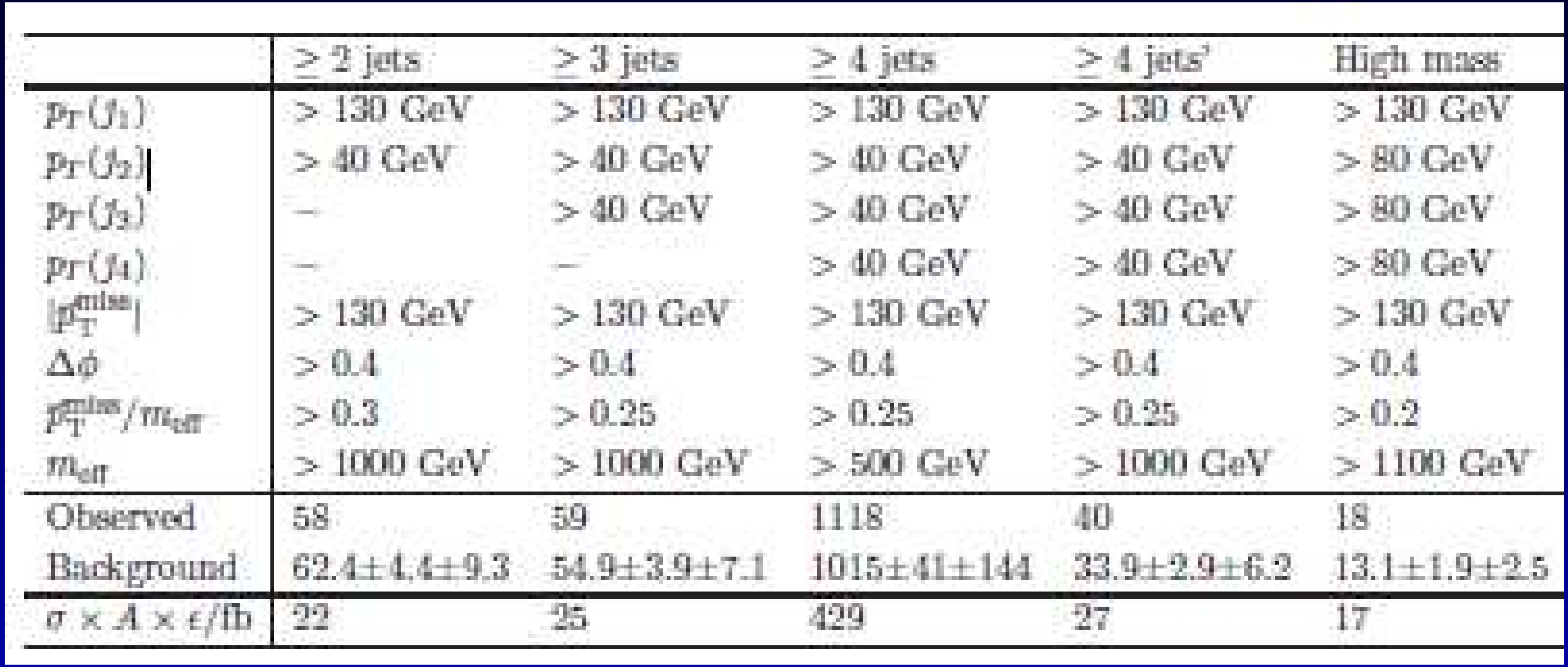
ATLAS 0-lepton, jets and $\not{p}_T$ search

ATLAS use cuts on different variables to search for SUSY:

- jet p_Ts
- $m_{eff} = \sum p_T^{(j)} + |\not{p}_T|$
- $m_T^{(i)^2}(\mathbf{p}_T{}^{(i)}, \not{\mathbf{q}}_T{}^{(i)}) \equiv 2\left|\mathbf{p}_T{}^{(i)}\right|\left|\not{\mathbf{q}}_T{}^{(i)}\right| - 2\mathbf{p}_T{}^{(i)} \cdot \not{\mathbf{q}}_T{}^{(i)}$ where $\not{\mathbf{q}}_T{}^{(i)}$ is the transverse momentum of particle (i). For each event, it is a lower bound on $m(NLSP)$.

$$M_{T2}(\mathbf{p}_T{}^{(1)}, \mathbf{p}_T{}^{(2)}, \not{\mathbf{p}}_T) \equiv \min_{\sum \not{\mathbf{q}}_T = \not{\mathbf{p}}_T} \left\{ \max\left(m_T^{(1)}, m_T^{(2)} \right) \right\}$$

ATLAS 1 fb^{-1} 0-lepton Search Results



	≥ 2 jets	≥ 3 jets	≥ 4 jets	≥ 4 jets'	High mass
$p_T(j_1)$	> 130 GeV	> 130 GeV	> 130 GeV	> 130 GeV	> 130 GeV
$p_T(j_2)$	> 40 GeV	> 40 GeV	> 40 GeV	> 40 GeV	> 80 GeV
$p_T(j_3)$	–	> 40 GeV	> 40 GeV	> 40 GeV	> 80 GeV
$p_T(j_4)$	–	–	> 40 GeV	> 40 GeV	> 80 GeV
$\lvert p_T^{\rm miss}\rvert$	> 130 GeV	> 130 GeV	> 130 GeV	> 130 GeV	> 130 GeV
$\Delta\phi$	> 0.4	> 0.4	> 0.4	> 0.4	> 0.4
$p_T^{\rm miss}/m_{\rm eff}$	> 0.3	> 0.25	> 0.25	> 0.25	> 0.2
$m_{\rm eff}$	> 1000 GeV	> 1000 GeV	> 500 GeV	> 1000 GeV	> 1100 GeV
Observed	58	59	1118	40	18
Background	$62.4\pm4.4\pm9.3$	$54.9\pm3.9\pm7.1$	$1015\pm41\pm144$	$33.9\pm2.9\pm6.2$	$13.1\pm1.9\pm2.5$
$\sigma \times A \times \epsilon$/fb	22	25	429	27	17

At any point in parameter space, one chooses the set of cuts with the greatest expected sensitivity[a].

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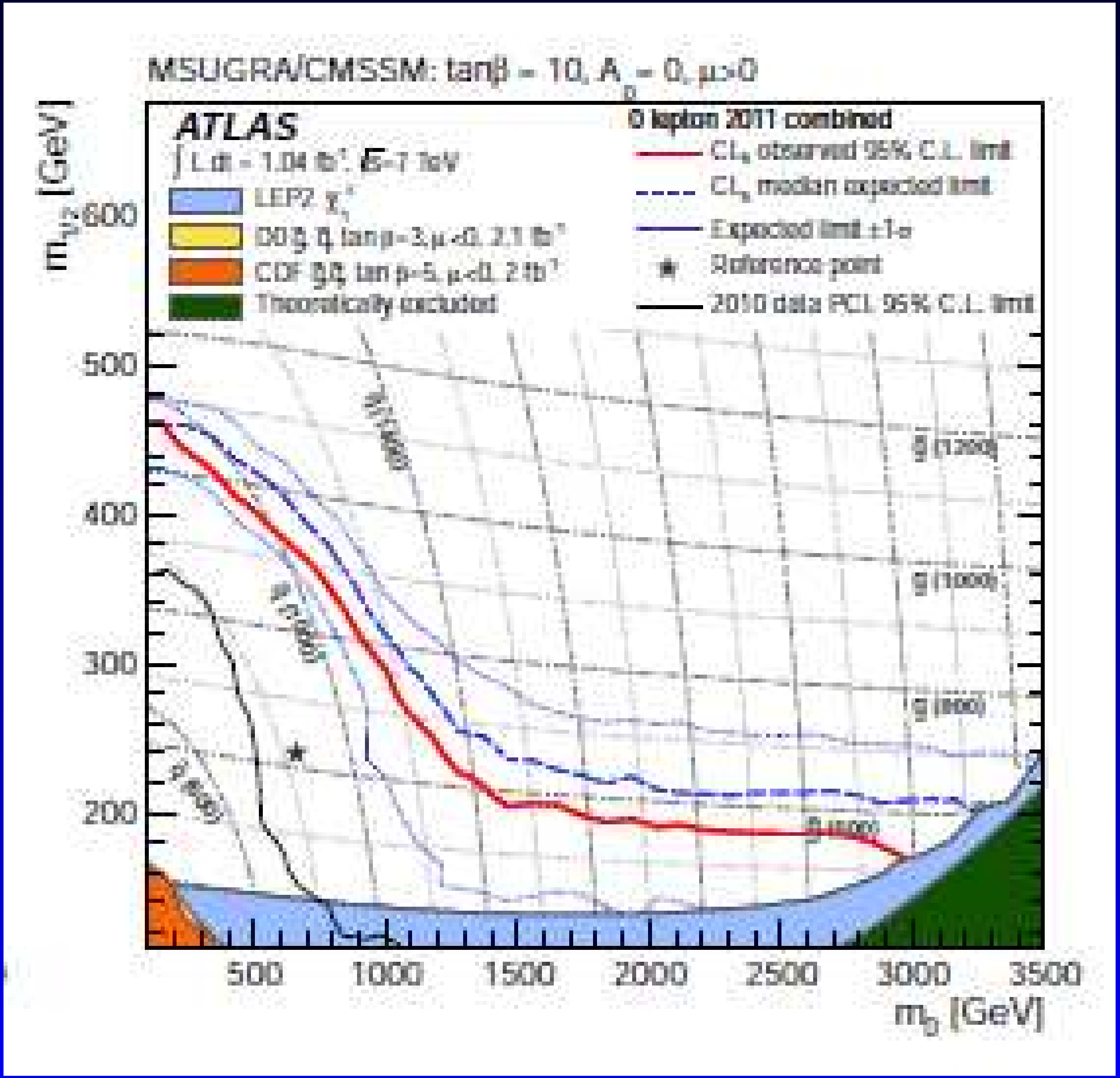
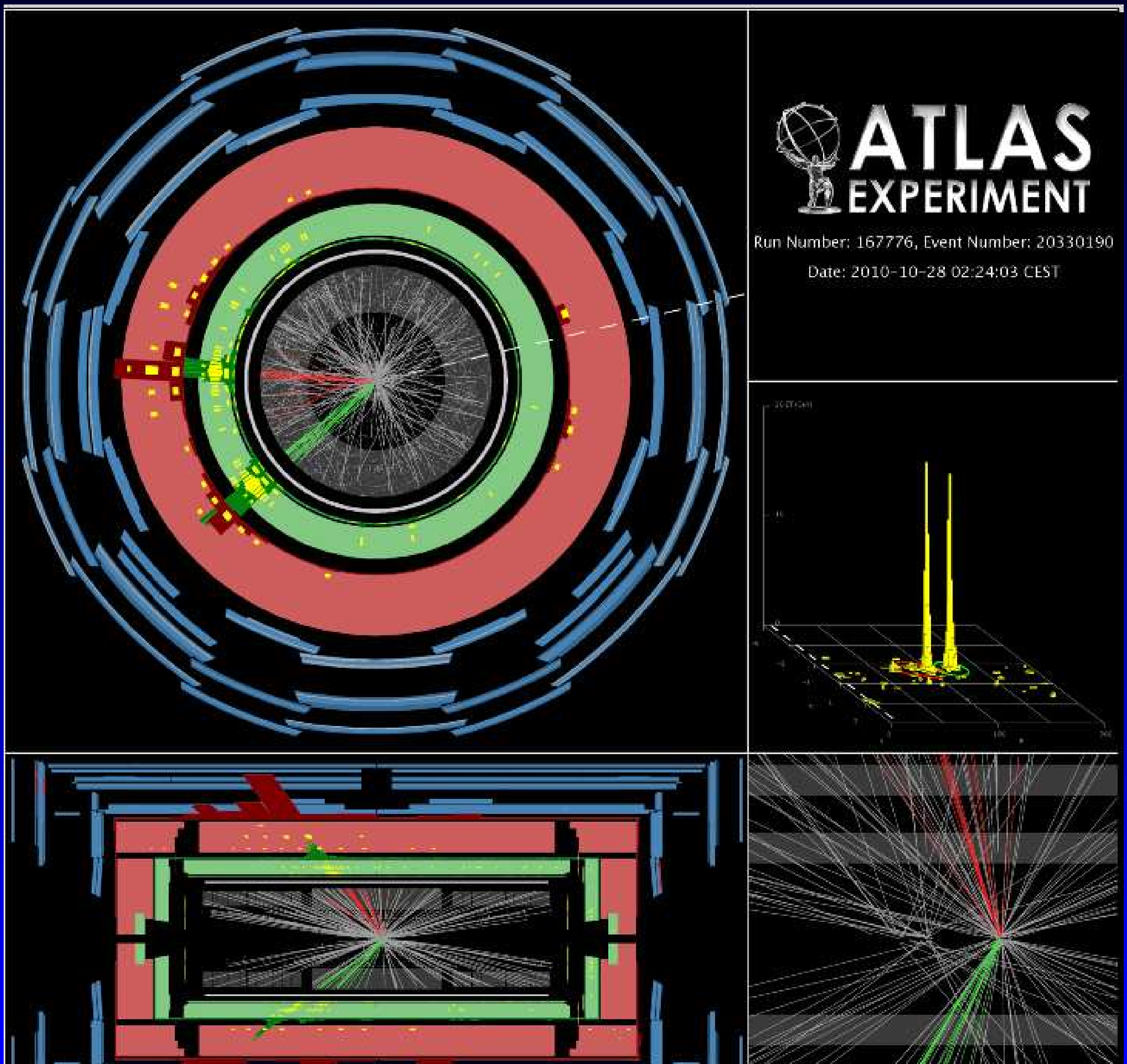
[a] ATLAS, `arxiv:1109.6572`

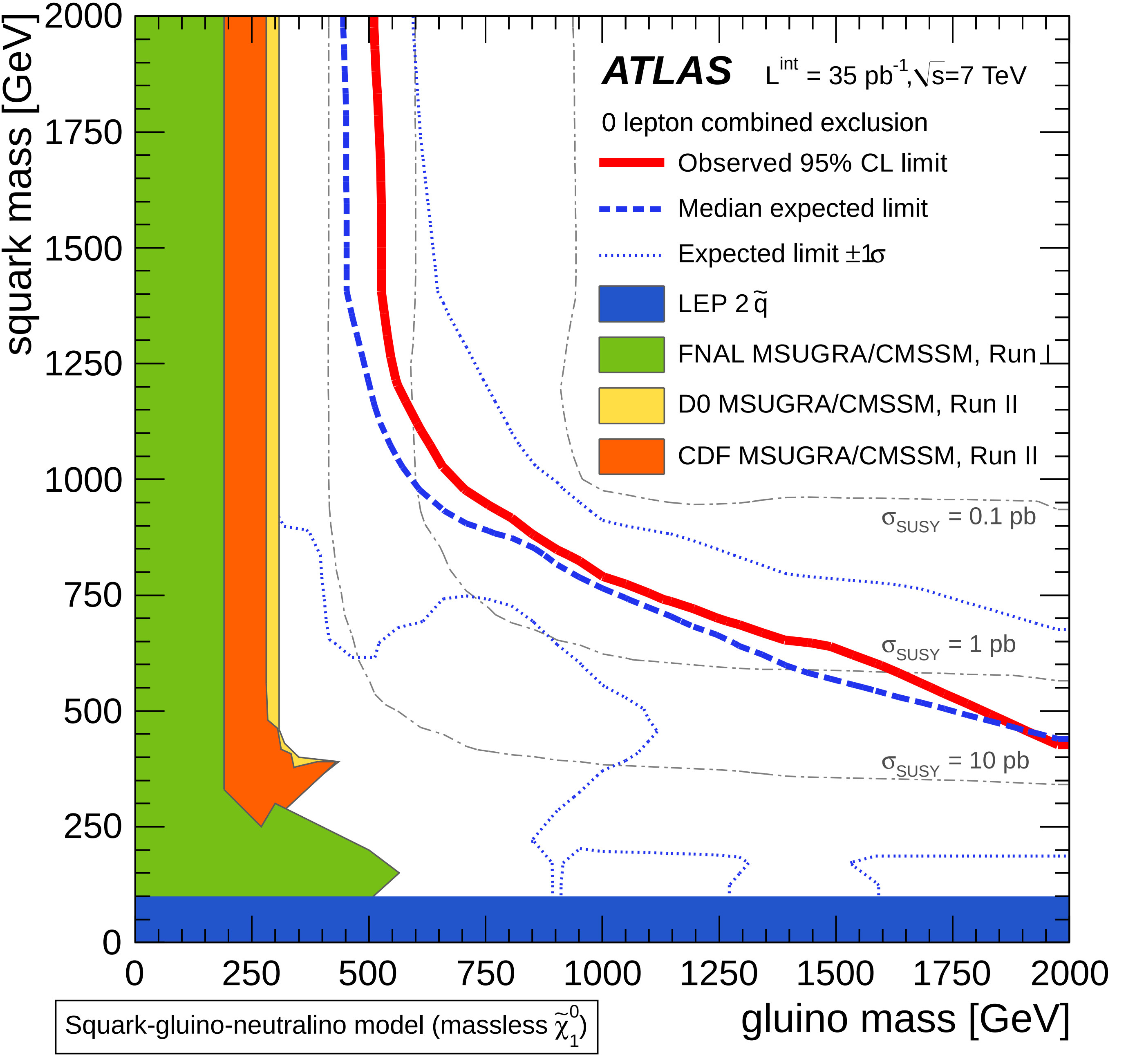
0-lepton CMSSM Exclusion



Candidate Event: High $E_T(j)$



MSSM Exclusion: Simplified Model



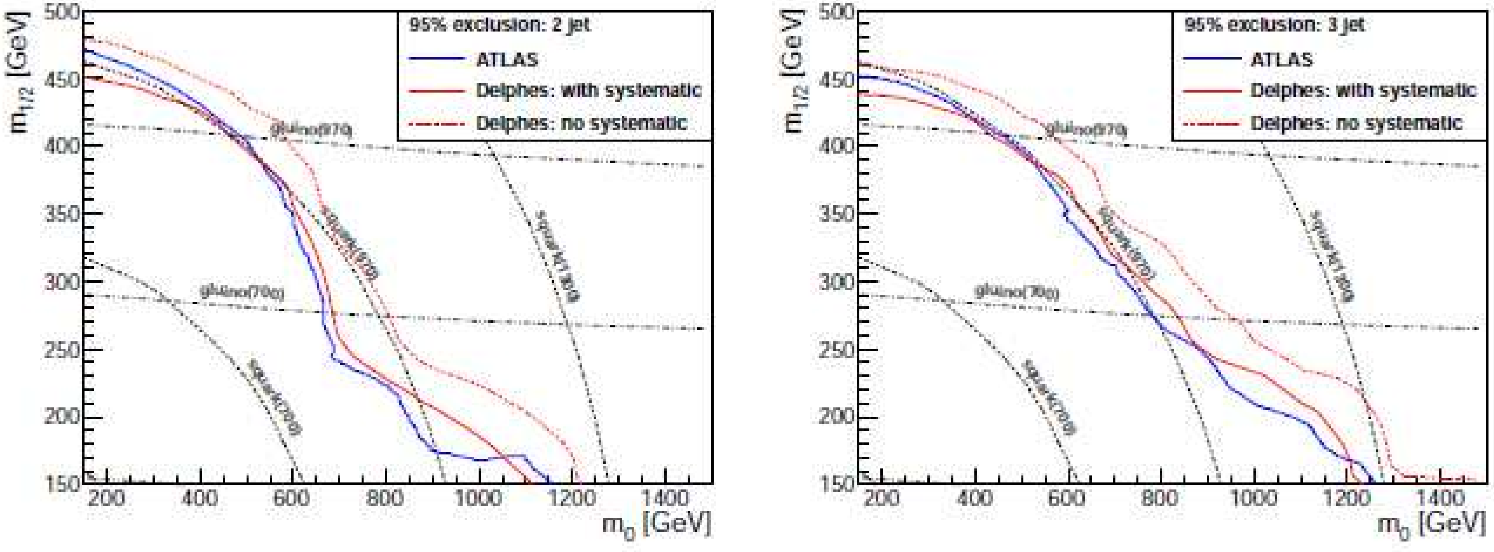
Intepretation

The results give a lower limit of 1020 GeV for $m_{\tilde{g}} = m_{\tilde{q}}$ in the CMSSM. We wish to *reinterpret* the search in mAMSB, to find the exclusion there (and study if mAMSB evades the search).

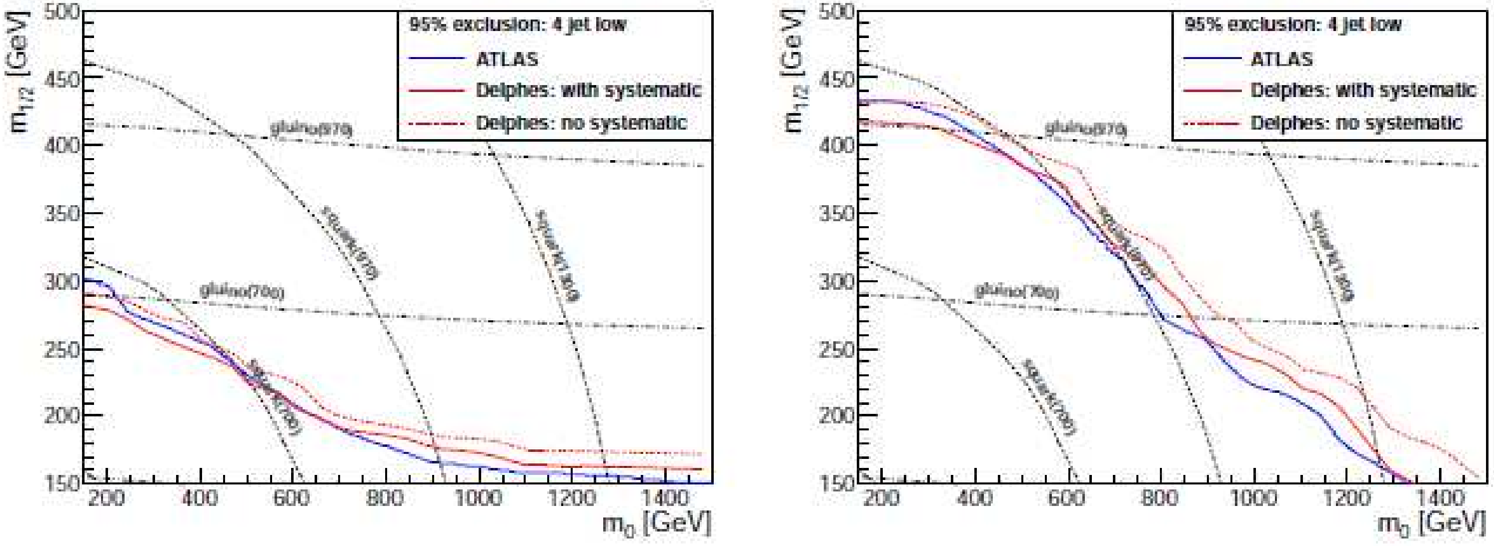
We simulate *signal* only, with `HERWIG++-2.5.1`, and use ATLAS' upper limits on $\sigma \times A \times \epsilon$. However *we have to fit the signal systematics*.

This becomes more involved when you want to do a fit and reconstruct the likelihood. To validate then, you need also details on the statistics.

ATLAS Validation



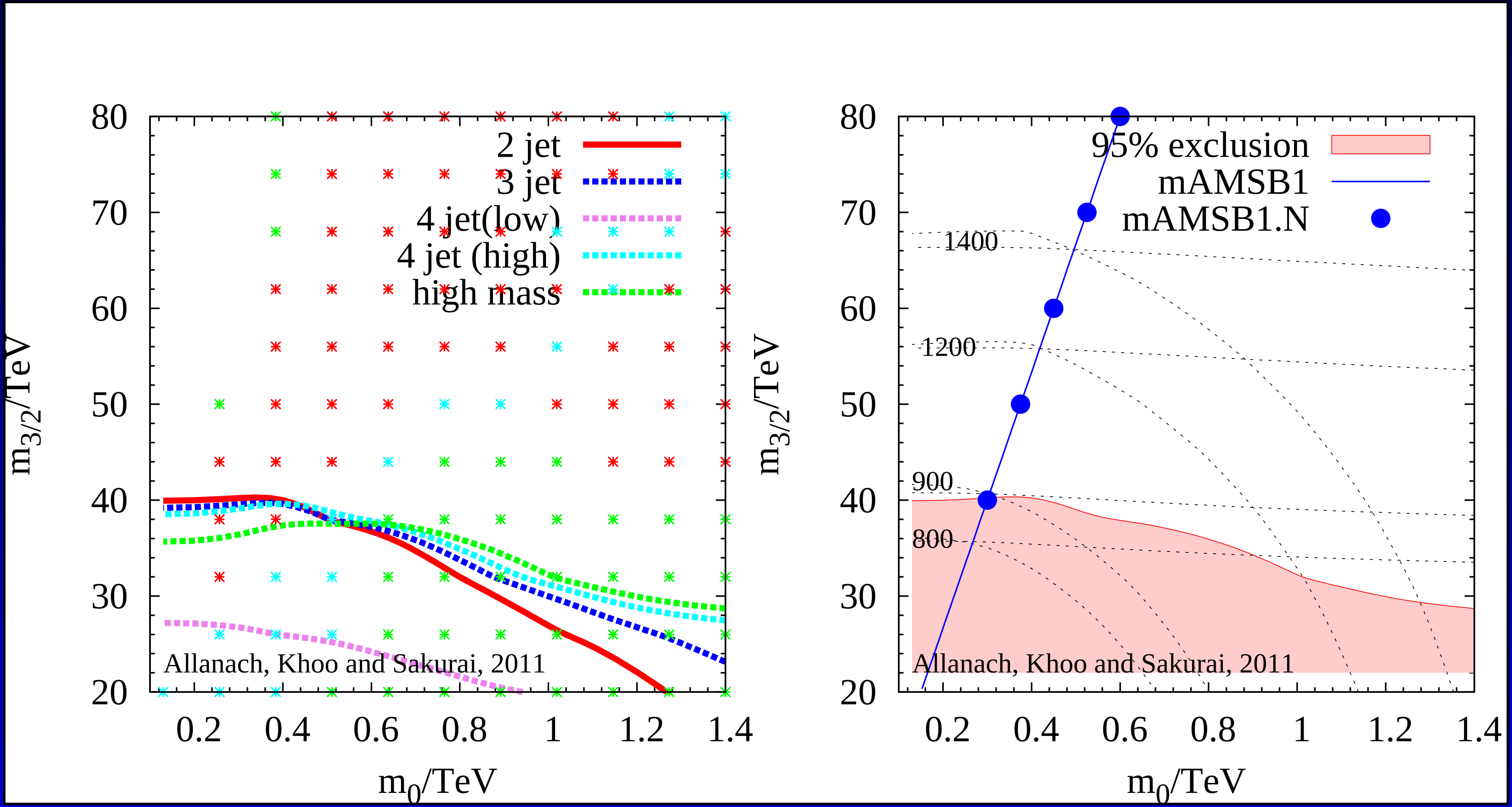
(a) 2 jets (b) 3 jets



(c) 4 jets (d) 4 jets'

mAMSB Exclusion

Interpret ATLAS exclusion in a different model: mAMSB.



Natural SUSY

The particles coupling the most strongly to the higgs are the *stops*[a]. Minimising the MSSM Higgs potential,

$$-\frac{M_Z^2}{2} = |\mu|^2 + m_{H_2}^2,$$

$$\delta m_{H_2}^2 = \frac{-3h_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{\Lambda_{UV}}{m_{\tilde{t}}}\right)$$

Applying that there should be no cancellation implies that

$$m_{\tilde{t}} \lesssim 700 \text{ GeV}, \qquad m_{\tilde{g}} \lesssim 1000 \text{ GeV}.$$

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[a] M. Papucci, J. T. Ruderman and A. Weiler, `arXiv:1110.6926`; G. Brust, A. Katz, S. Lawrence and R. Sundrum, `arXiv:1110.6670`

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$$\begin{aligned} -\frac{M_Z^2}{2} &= |\mu|^2 + m_{H_2}^2, \\ \delta m_{H_2}^2 &= \frac{-3h_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{\Lambda_{UV}}{m_{\tilde{t}}}\right) \end{aligned}$$

Experimental $\not{E}_T$ searches currently rule out

$$m_{\tilde{t}} \stackrel{<}{\sim} 500 \text{ GeV}, \qquad m_{\tilde{g}} \stackrel{<}{\sim} 800 \text{ GeV}.$$

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[a] M. Papucci, J. T. Ruderman and A. Weiler, `arXiv:1110.6926`; C. Brust, A. Katz, S. Lawrence and R. Sundrum, `arXiv:1110.6670`

Definition of R-Parity

$\mathcal{Q}$: How is $W_{\not R_P}$ normally banned?
$\mathcal{A}$: By defining discrete symmetry R_p

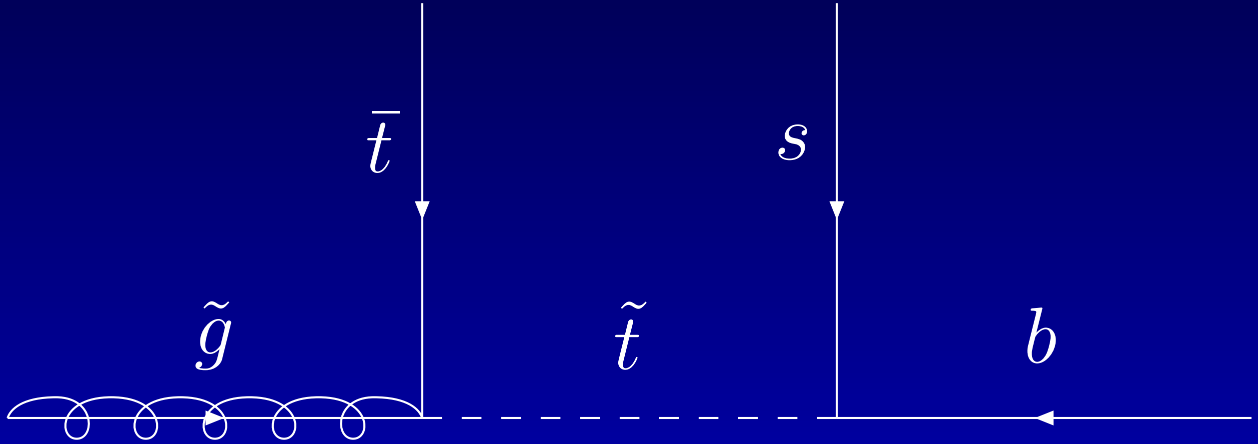
$$R_p = (-\mathbf{1})^{3B+L+2S}.$$

$\rightarrow$ SM fields have $R_p = +1$ and superpartners have $R_p = -1$. There are two important consequences:

- Because initial states in colliders are R_p EVEN, we can only pair produce SUSY particles
- The *lightest superpartner is stable*

Baryon Number Violation

Can lead to natural SUSY with light stops and gluinos that hasn't been excluded yet *because there is no large $\not{E}_T$ signature*[a].



$\tilde{g}\tilde{g}$ production dominates: we have some ideas on how to reinterpret current *non-SUSY* searches in terms of this model.

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[a] BCA and Ben Gripaios, `arXiv:1202.6616`

Shopping List

Things that the CMS/ATLAS always provide that we need:

- Cuts and numbers of events observed past them
- Expected background numbers with systematic errors

We could really do with:

- Keeping in mind: we can't combine analyses that use the same events: much better to keep the events disjoint. Doesn't preclude fully inclusive analysis, but make the others as disjoint as possible.
- Likelihood versus predicted number of events past cuts (before efficiency correction). Ideally, sanitized `RooStats`

Shopping List II

Failing that, then we must calculate the likelihood:

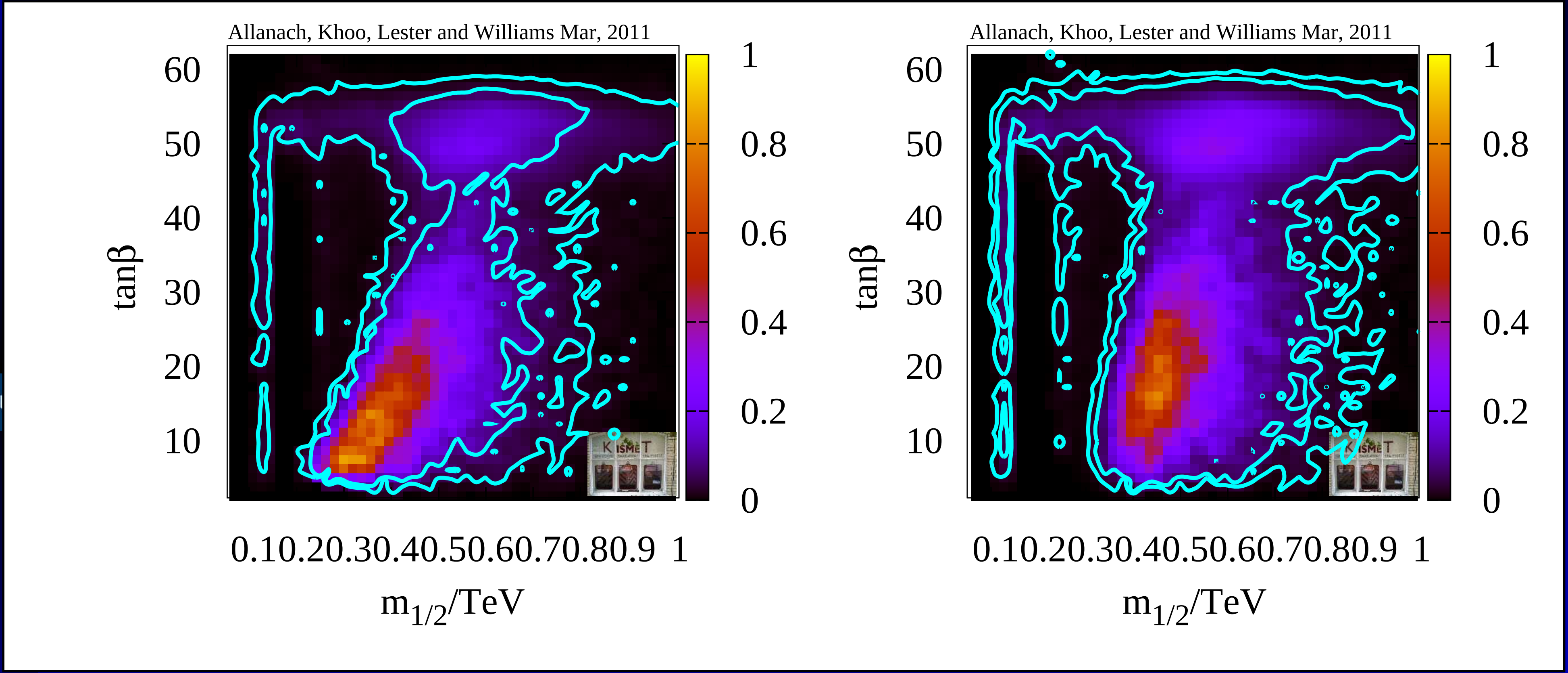
- Systematic errors on signals: perhaps at least a range over parameter space in one model. Ideally, it would be parameterised in terms of important quantities.
- Other contours (eg 1/5 sigma exclusion contours) so we can check our likelihood away from 95% excluded region.
- Numbers in histogram plots attached to `arXiv` publication

Summary

- LHC has ruled out a significant portion of the most natural part of SUSY parameter space, but will require *nothing* at 14 TeV to dissuade most fans.
- Constrained models are now getting even more squeezed. Experiments are inviting theorists to do the interpretation in terms of their favourite SUSY breaking model.
- Current *missing energy type* searches reach squark and gluino masses of 1020 GeV (CMSSM), 900 GeV (mAMSB). Too early to give up on SUSY though.
- Another possibility: we haven't seen SUSY yet because $R-$parity is violated

Supplementary Material

Log Fits

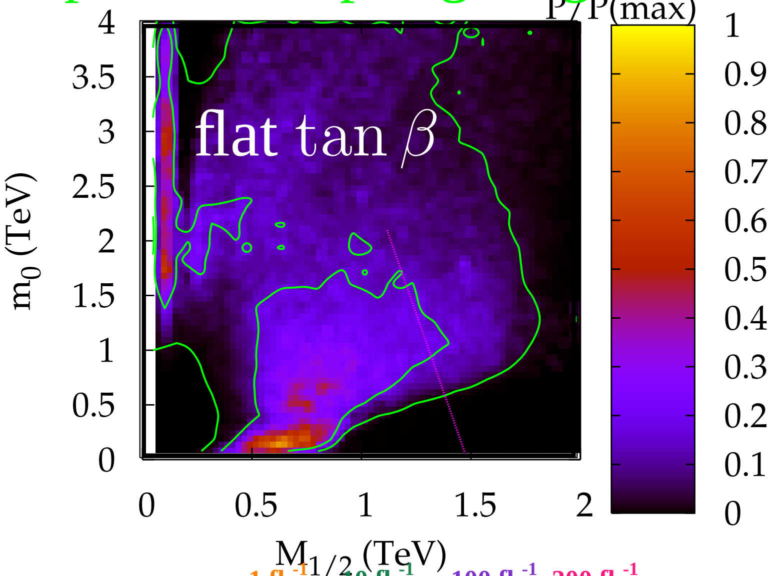


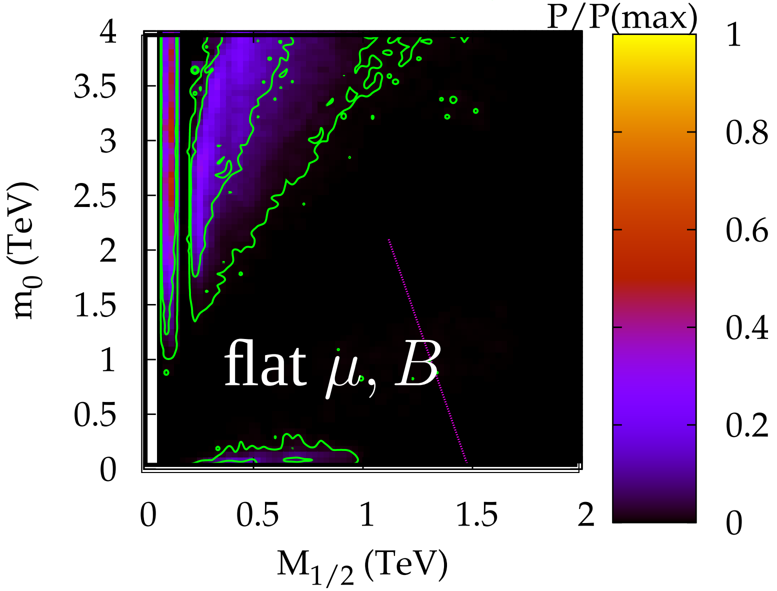
Before (left) and after (right) ATLAS 0-lepton exclusion limits.

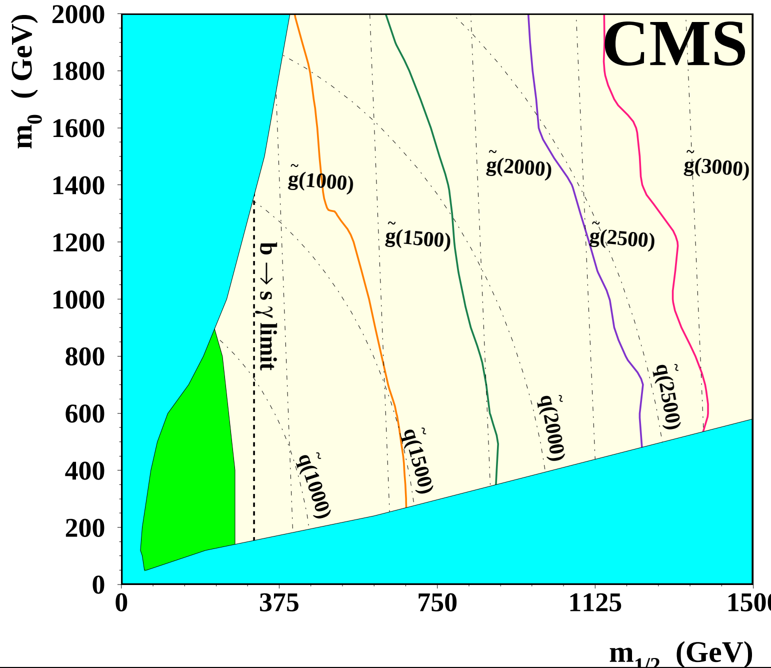
Killer Inference for Susy METeorology

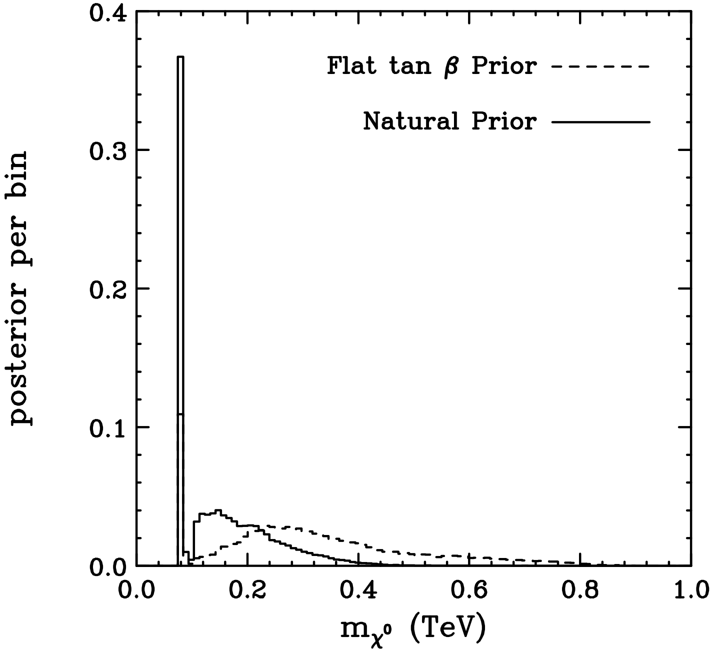
BCA, Cranmer, Weber, Lester, `arXiv:0705.0487`

http://users.hepforge.org/~allanach/benchmarks/kismet.html



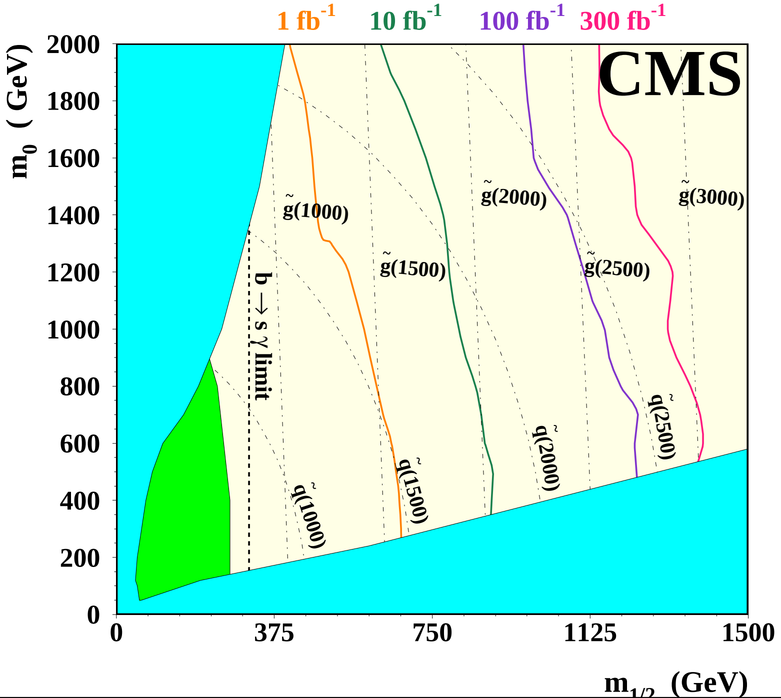


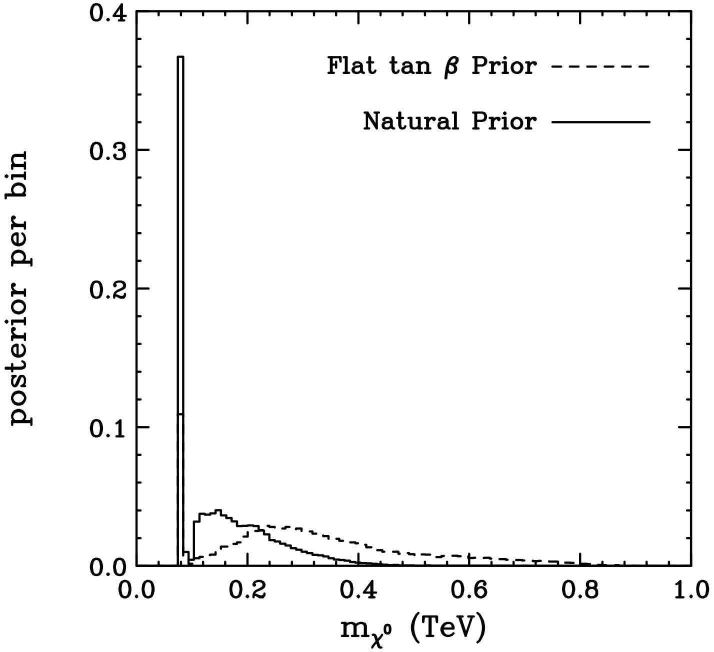




Killer Inference for Susy METeorology

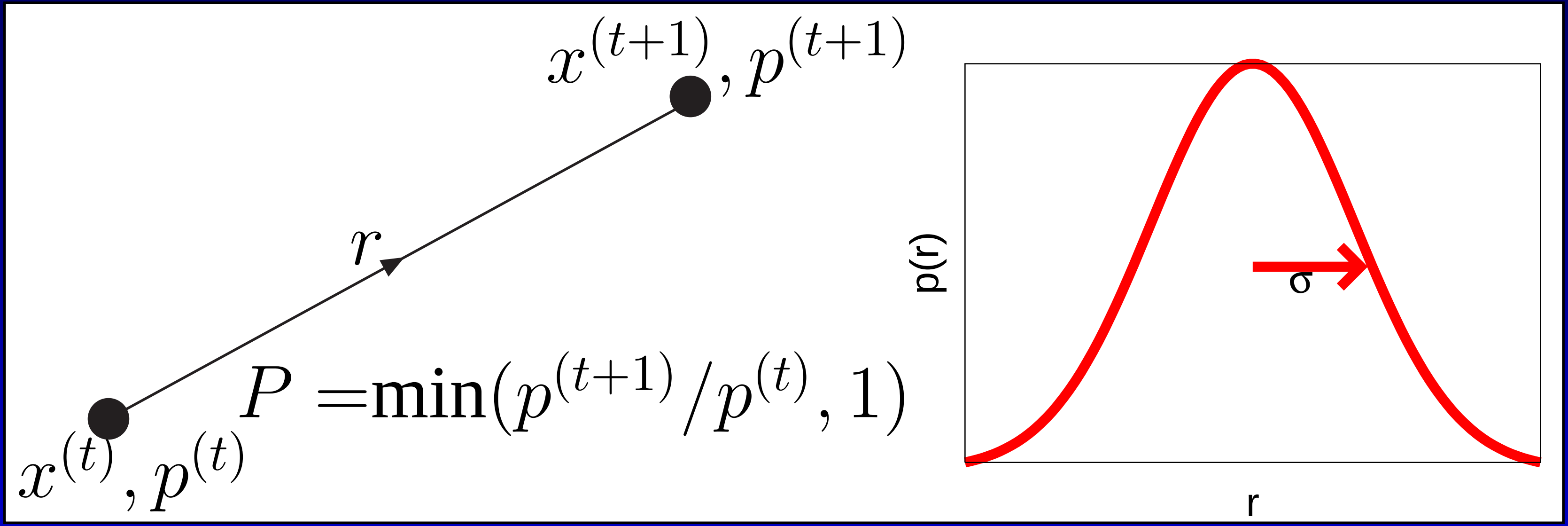
BCA, Cranmer, Weber, Lester, `arXiv:0705.0487`





Markov-Chain Monte Carlo

Metropolis-Hastings Markov chain sampling consists of list of parameter points $x^{(t)}$ and associated posterior probabilities $p^{(t)}$.



Final density of x points $\propto p$. Required number of points goes *linearly* with number of dimensions.

Ice Cube

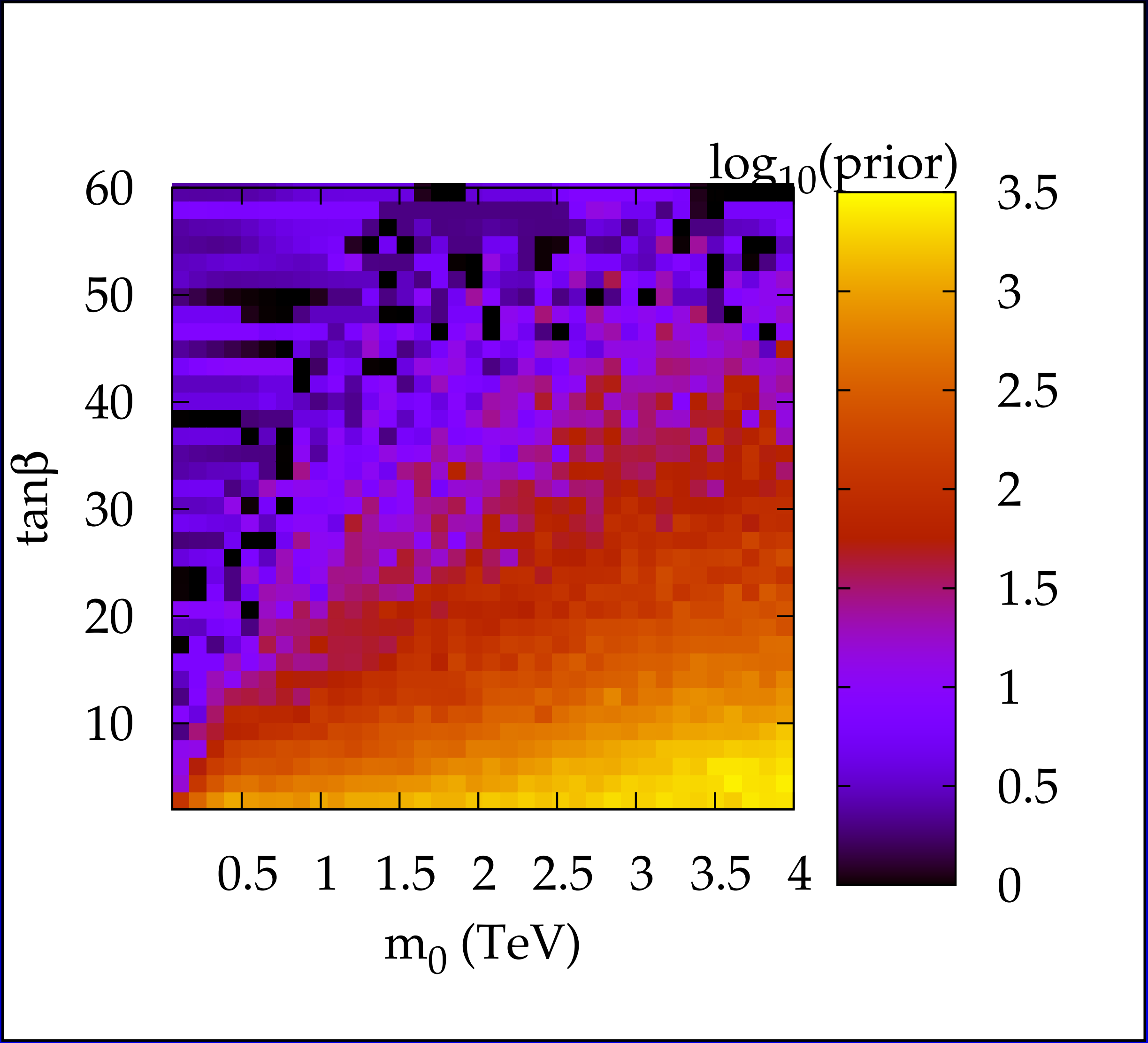
Neutralinos can become trapped in the sun $\tilde{h}^0 - Z$ coupling $\sigma_{\chi^0 \text{p,SD}} \propto [|N_{1d}|^2 - |N_{1u}|^2]^2$ dominates.
$A^\odot \equiv \sigma v / V$:

$$\begin{aligned} \dot{N} &= C^\odot - A^\odot N^2, \\ \Gamma &= \frac{1}{2} A^\odot N^2 = \frac{1}{2} C^\odot \tanh^2 \left(\sqrt{C^\odot A^\odot} t_\odot \right) \\ \frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} &= \frac{C_\odot F_{\text{Eq}}}{4\pi D_{\text{ES}}^2} \left(\frac{dN_\nu}{dE_\nu} \right)^{\text{Inj}} \\ N_{\text{ev}} &\approx \int \int \frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} \frac{d\sigma_\nu}{dy} R_\mu((1-y) E_\nu) A_{\text{eff}} \, dE_{\nu_\mu} \, dy \end{aligned}$$

Science & Technology Facilities Council

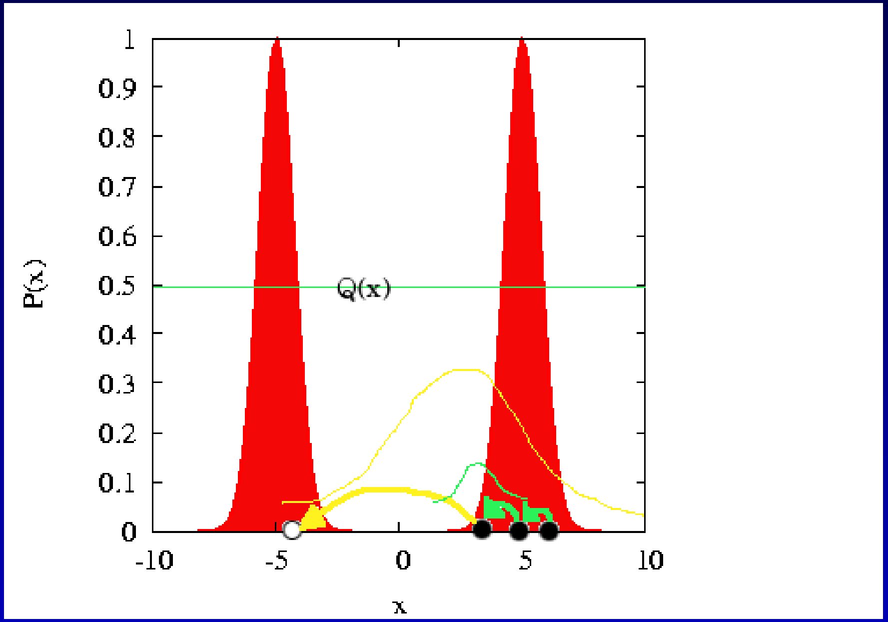
Supersymmetry Cambridge Working group

Naturalness priors



Potential Problem

Often, people use a flat $Q(x)$. The trouble with this *“blind drunk”* sampling is the following situation:



Either large or small proposal widths σ lead to low efficiencies of sampling. Our proposal is to determine a $Q(x)$ closer to $P(x)$ *semi-automatically*.

Bank Sampling

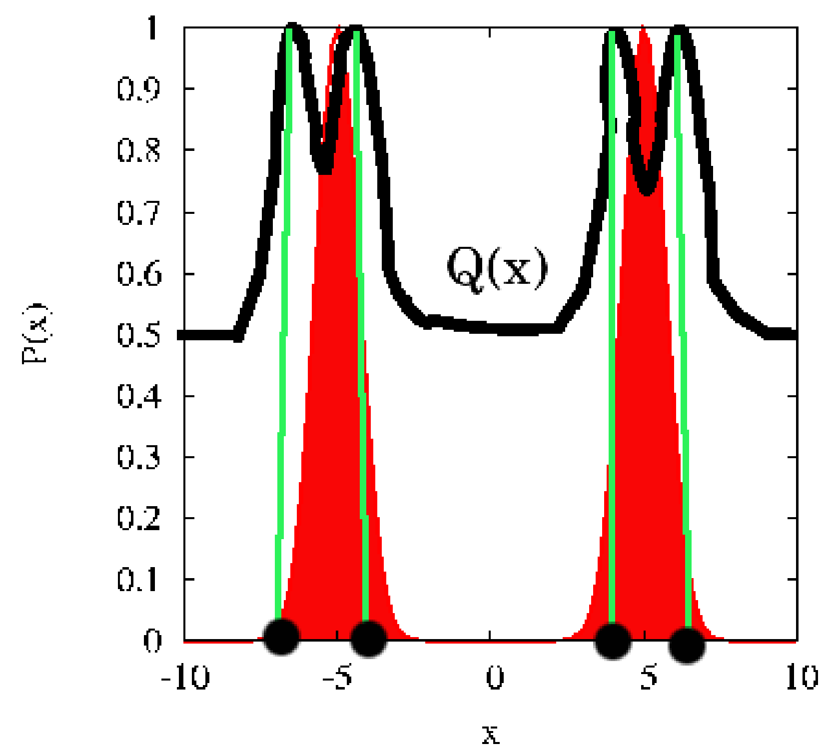


Figure 5: Bank points determined from previous runs: want to have at least one point in each maximum. *Knowledgeable drunk*

Proposal Distribution

$$Q_{bank}(\mathbf{x};\mathbf{x}^{(t)}) = (1-\lambda)K(\mathbf{x};\mathbf{x}^{(t)})+\lambda\sum_{i=1}^{N} w_i K(\mathbf{x};\mathbf{y}^{(i)})$$

w_i are a set of N weights: $\sum_{i=1}^{N} w_i = 1$, $0 < \lambda < 1$, while K is the proposal distribution.

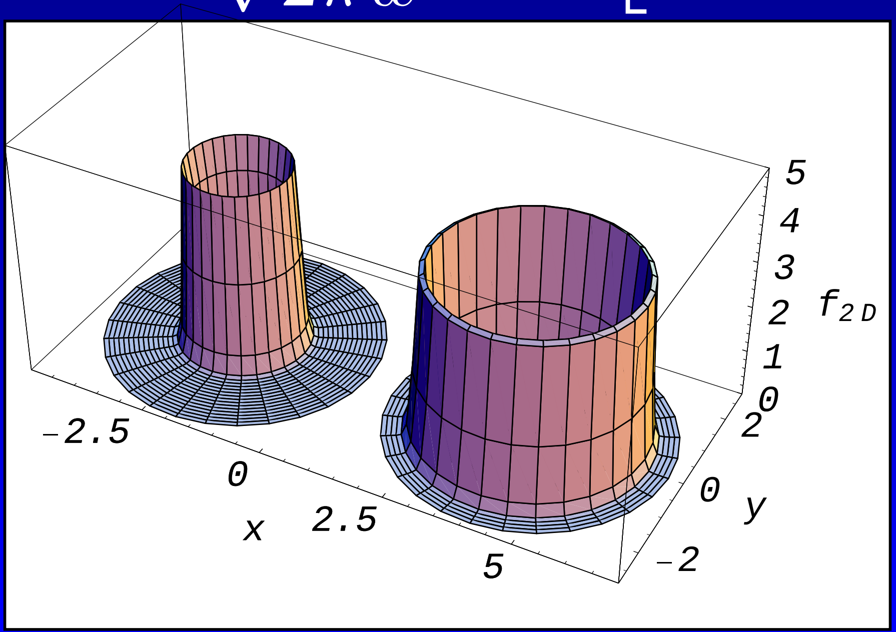
With probability $(1-\lambda)$ propose a local Metropolis step of the usual kind, i.e. “close” to the last point in the chain. With probability λ, teleport to the vicinity of one of the number of “banked” points, chosen with weight w_i from within the bank.

Example Distribution

$$f_{2D}(\mathbf{x}) = \mathrm{circ}(\mathbf{x}; c_1, r_1, w_1) + \mathrm{circ}(\mathbf{x}; c_2, r_2, w_2)$$

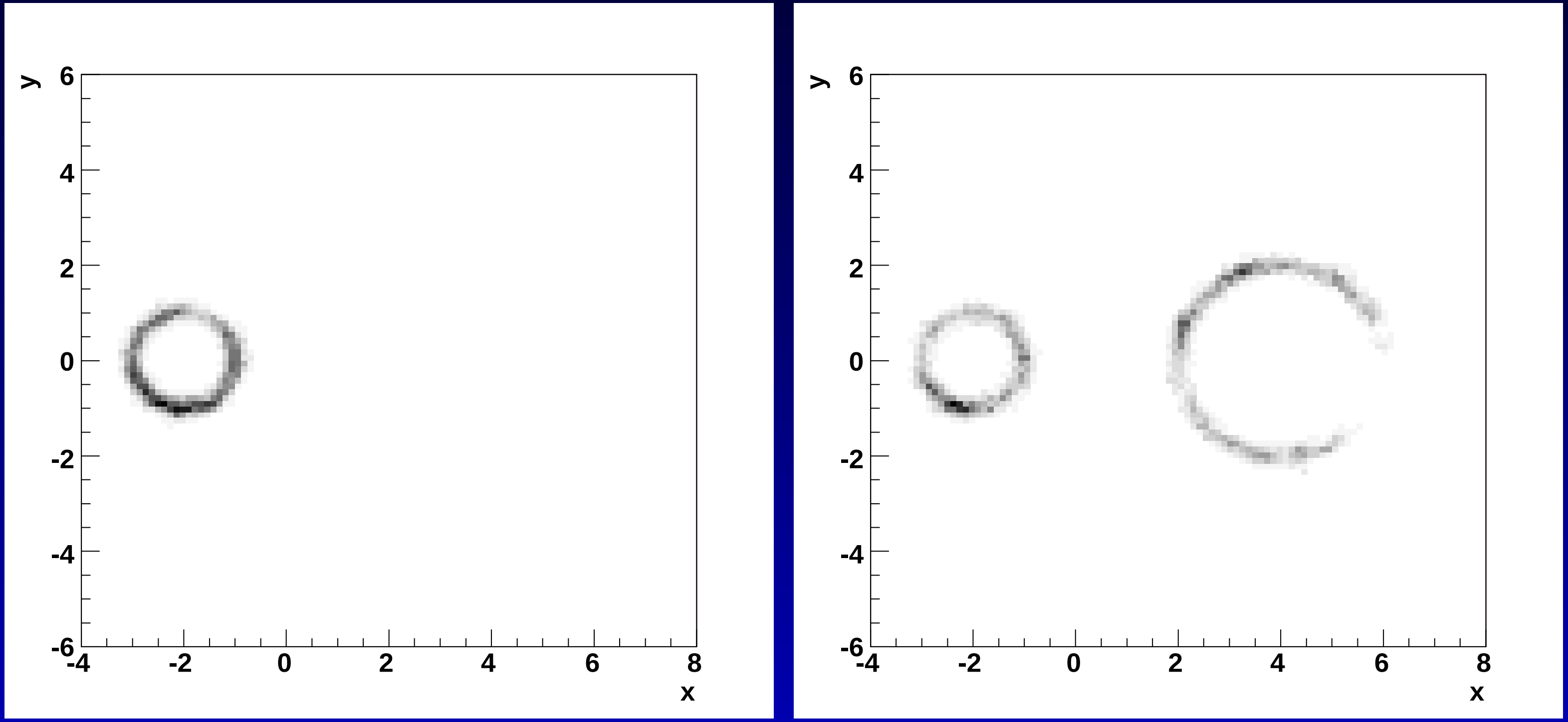
where $c_1 = (-2, 0)$, $r_1 = 1$, $w_1 = 0.1$, $c_2(+4, 0)$, $r_2 = 2$, $w_2 = 0.1$ and

$$\mathrm{circ}(\mathbf{x}; \mathbf{c}, r, w) = \frac{1}{\sqrt{2\pi w^2}} \exp\left[-\frac{(|\mathbf{x} - \mathbf{c}| - r)^2}{2w^2} \right].$$



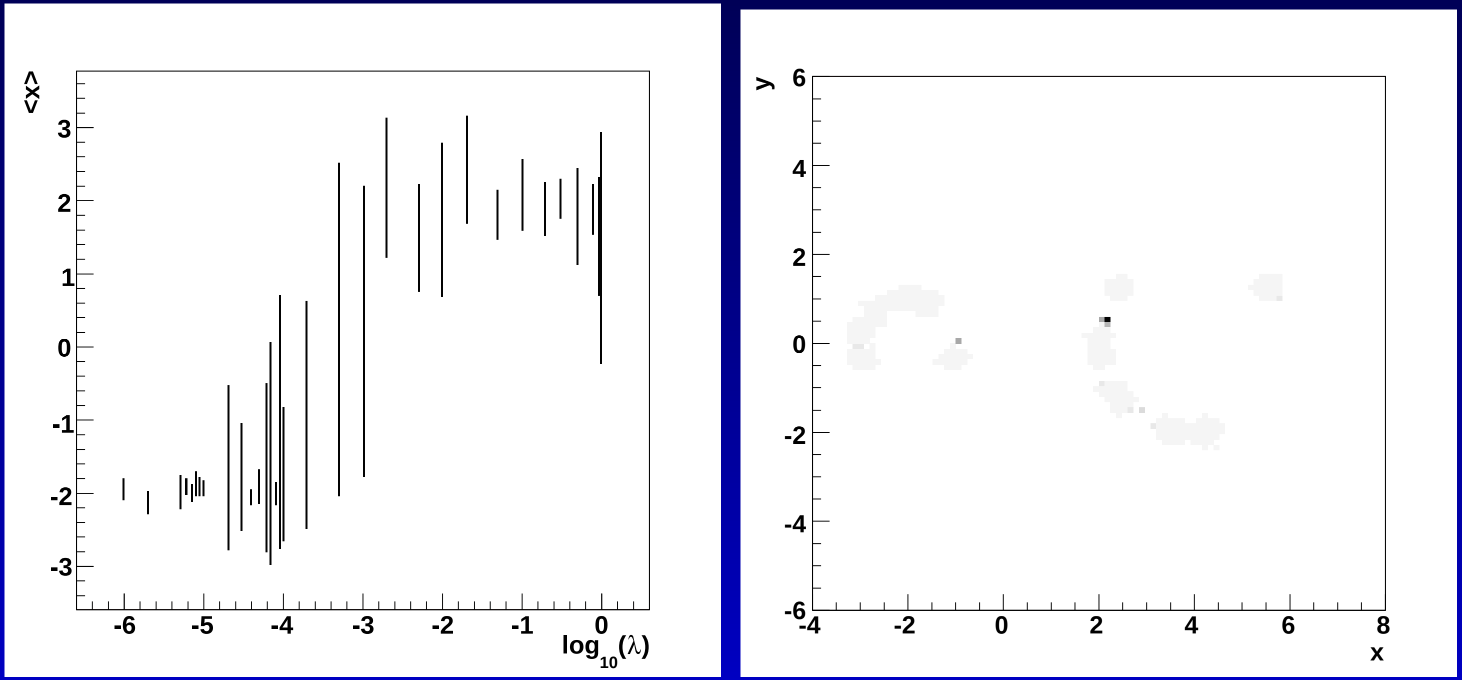
Bank vs Metropolis

10 000 samples for MCMC and bank sampling:



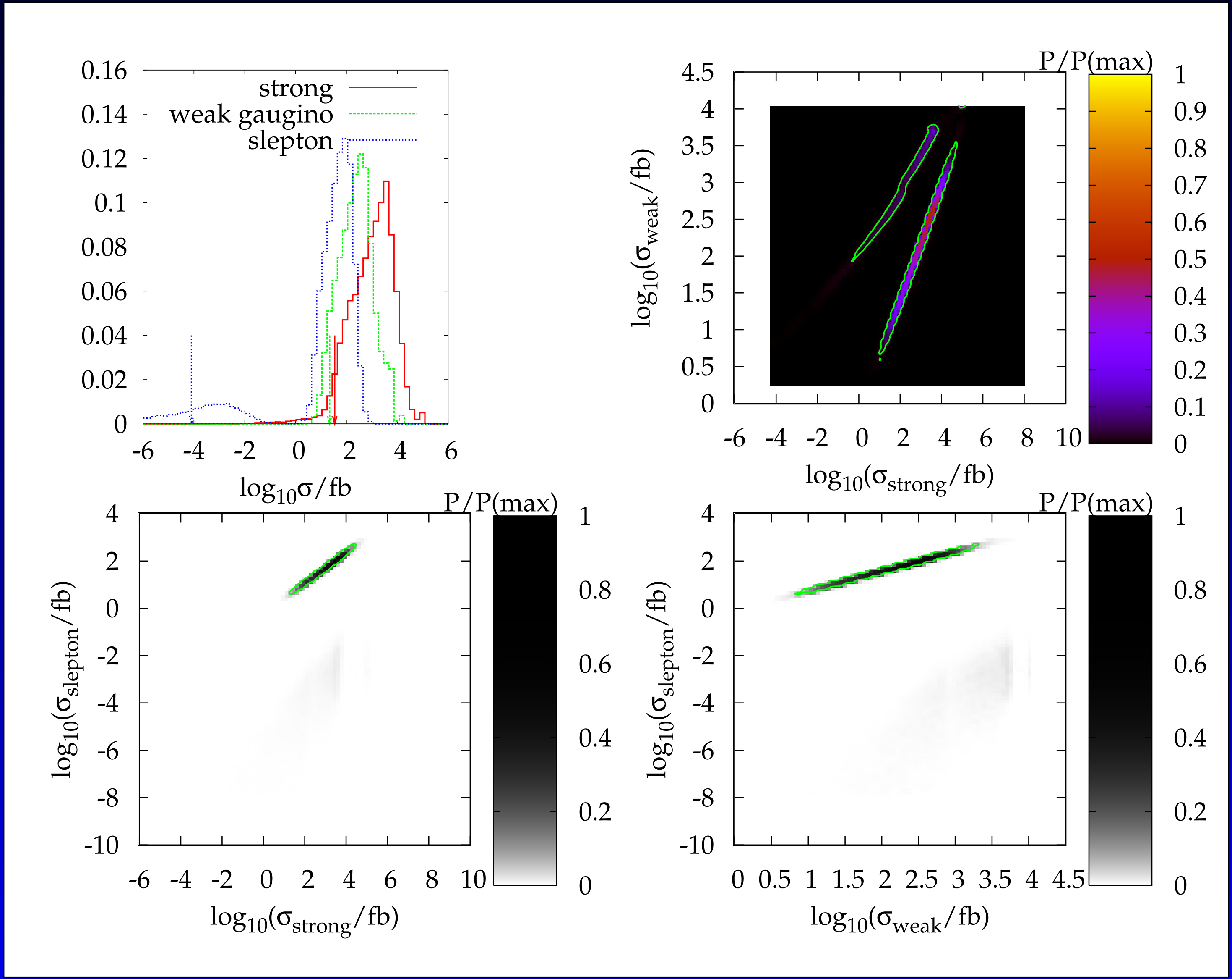
Safety with respect to λ

10 bank samplers, with 10 bank points generated in each circle: 10 000 samples. All started from $x = -2$. Correct $\langle x \rangle = 2$. $\lambda \approx 1$ is importance sampling limit.



$\mathcal{Q}$: What values of λ are “safe”?
$\mathcal{A}$: [0.001, 0.9]

LHC Cross-sections



Collider Check

Need corroboration with *direct detection*.

If we can pin particle physics down, a comparison between the predicted relic density and that observed is a test of the cosmological assumptions used in the prediction.[a]

Thus, if it doesn't fit, you change the cosmology until it does.

---

[a] BCA, G. Belanger, F. Boudjema, A. Pukhov, JHEP 0412 (2004) 020.; M. Nojiri, D. Tovey, JHEP 0603 (2006) 063

CMSSM Regions

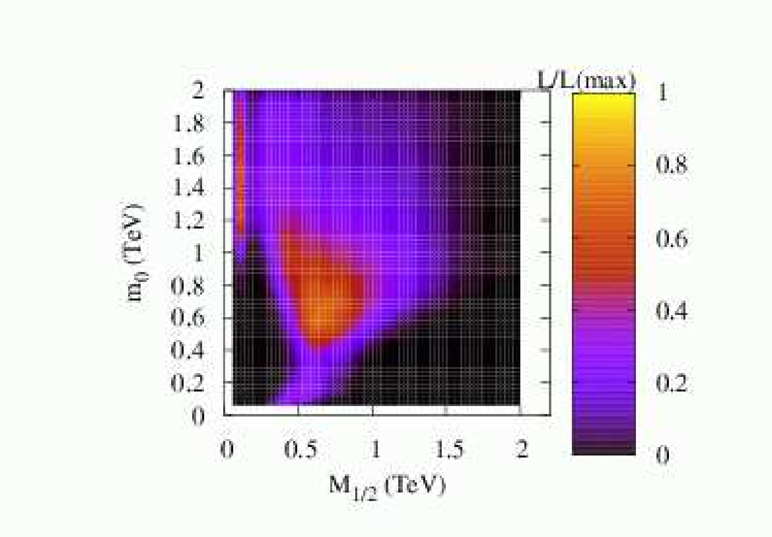
After WMAP+LEP2, bulk region diminished. Need specific mechanism to reduce overabundance:

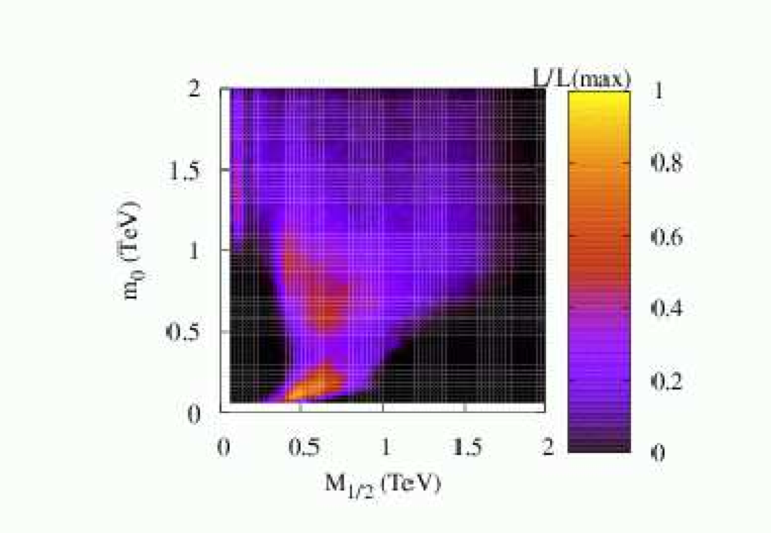
- $\tilde{\tau}$ coannihilation: small m_0, $m_{\tilde{\tau}_1} \approx m_{\chi_1^0}$. Boltzmann factor $\exp(-\Delta M/T_f)$ controls ratio of species. $\tilde{\tau}_1 \chi_1^0 \to \tau\gamma$, $\tilde{\tau}_1 \tilde{\tau}_1 \to \tau\bar{\tau}$.
- Higgs Funnel: $\chi_1^0 \chi_1^0 \to A \to b\bar{b}/\tau\bar{\tau}$ at large $\tan\beta$. Also via[a] h at large m_0 small $M_{1/2}$.
- Focus region: Higgsino LSP at large m_0: $\chi_1^0\chi_1^0 \to WW/ZZ/Zh/t\bar{t}$.
- $\tilde{t}$ coannihilation: high $-A_0$, $m_{\tilde{t}_1} \approx m_{\chi_1^0}$. $\tilde{t}_1\chi_1^0 \to gt$, $\tilde{t}\tilde{t} \to tt$

---

[a] Datta, Djouadi, Drees, `hep-ph/0504090`

Comparison



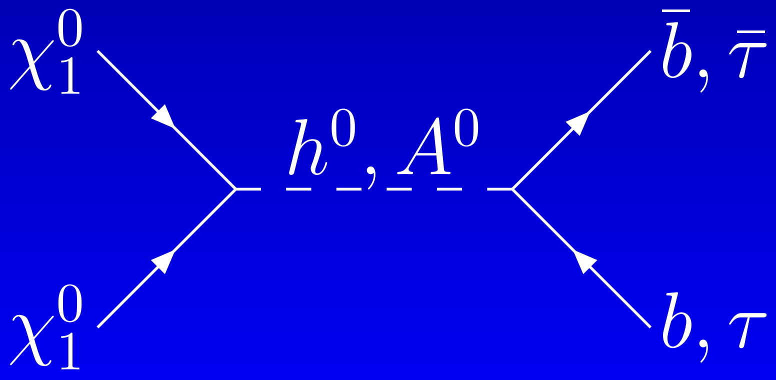


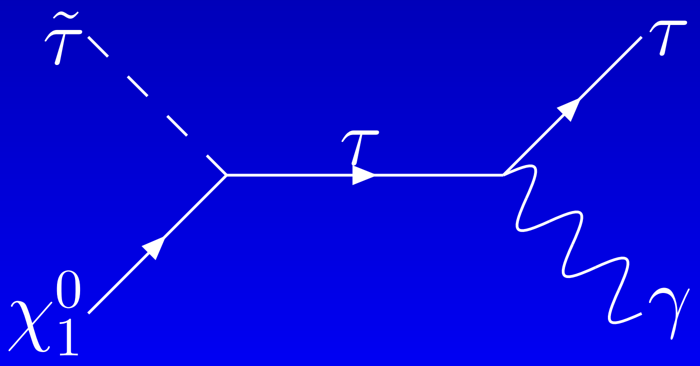
- LHS: allowing non thermal-χ_1^0 contribution
- RHS: only χ_1^0 dark matter
- *(flat priors)*

Annihilation Mechanism

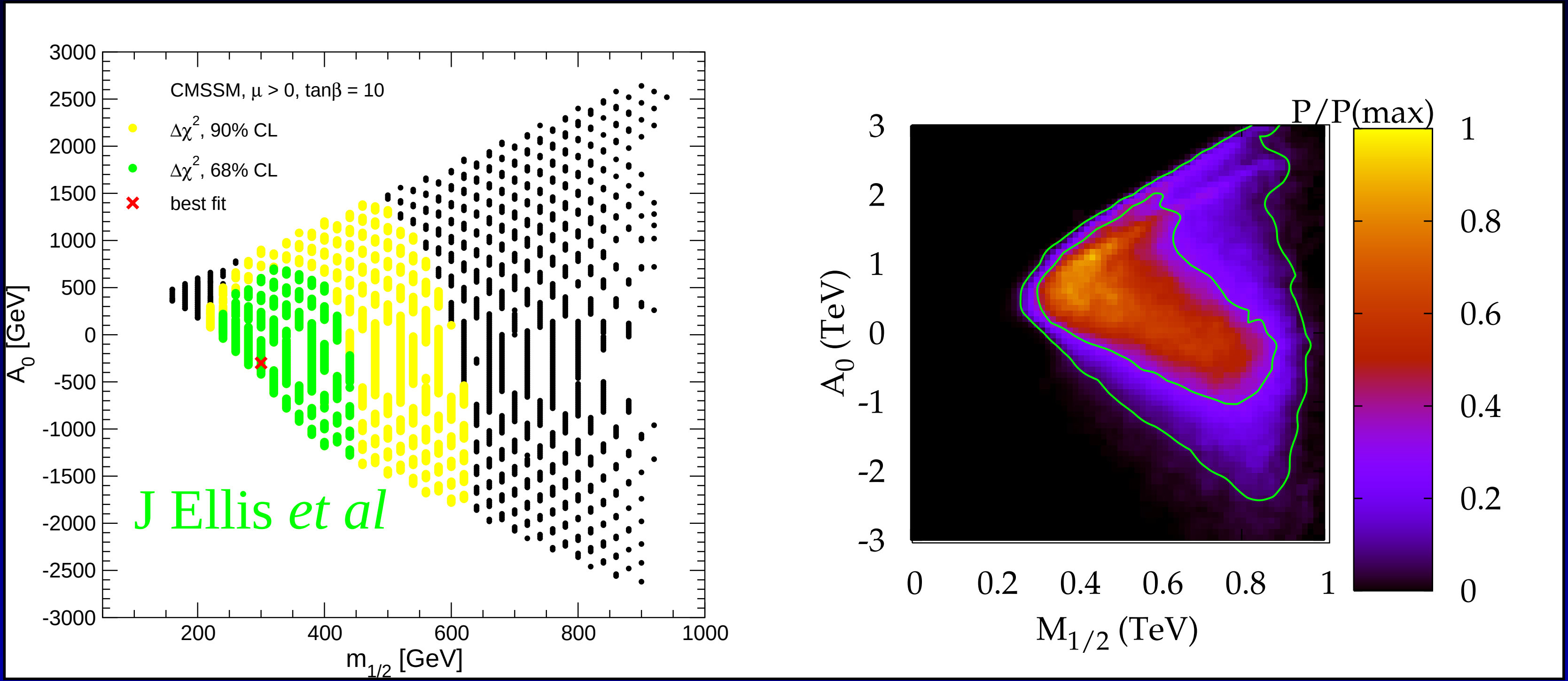
Define stau co-annihilation when $m_{\tilde{\tau}}$ is within 10% of $m_{\chi_1^0}$ and Higgs pole when $m_{h,A}$ is within 10% of $2m_{\chi_1^0}$.

mechanism	flat prior	natural prior
h^0-pole	0.025	0.07
A^0-pole	0.41	0.14
$\tilde{\tau}-$co-annihilation	0.26	0.18
rest	0.31	0.61



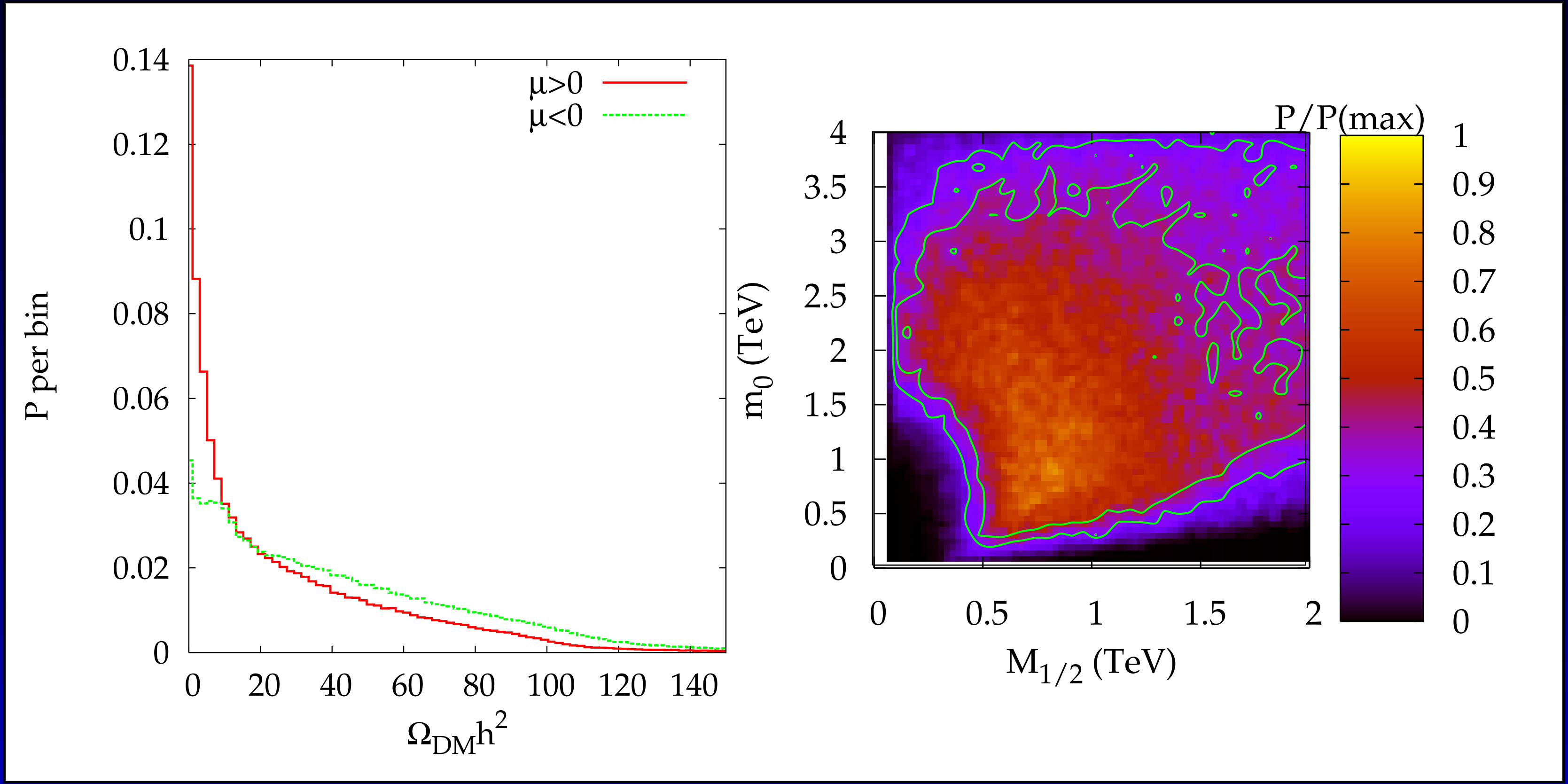


Comparison



- Fix $\tan\beta = 10$ and all SM inputs
- Restrict $m_0, M_{1/2} < 1$ TeV.
- *Same* fits!

No Dark Matter Fits



Huge χ^2 from the dark matter relic density.

Volume Effects

Can't rely on a good χ^2 in non-Gaussian situation

Likelihood and Posterior

Q: What’s the chance of observing someone to be pregnant, given that they are female?

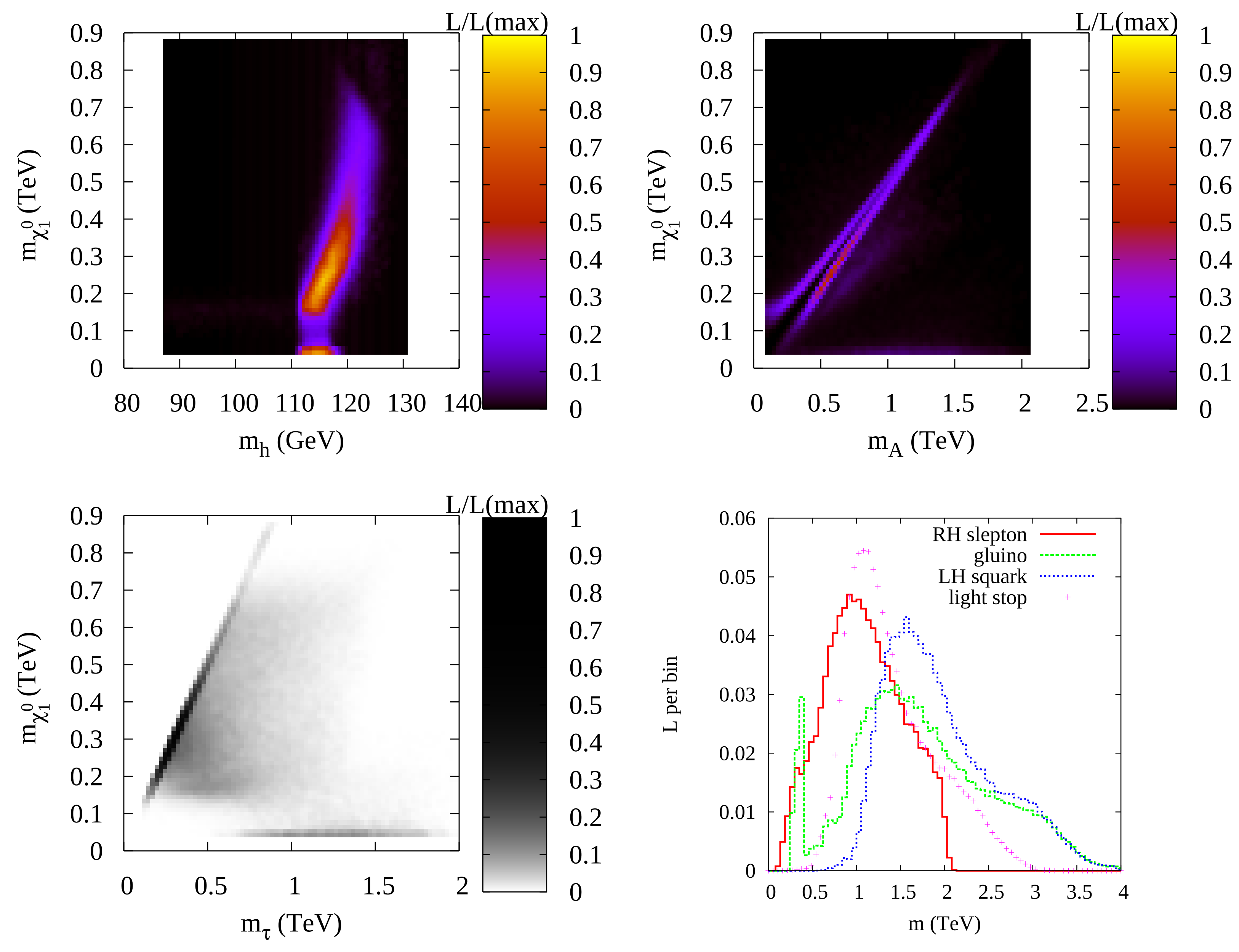
Likelihood

$p(\text{pregnant} \mid \text{female, human}) = 0.01$

Posterior

$p(\text{female} \mid \text{pregnant, human}) = 1.00$

Sanity Check



LHC vs LC in SUSY Measurement

- LHC (start date 2007) produces strongly interacting particles up to a few TeV. Precision measurements of mass *differences* possible if the decay chains exist: possibly per mille for leptons, several percent for jets.
- ILC has several energy options: 500-1000 GeV, CLIC up to 3 TeV. Linear colliders produce less strong particles but much easier to make precision measurements of masses/couplings.

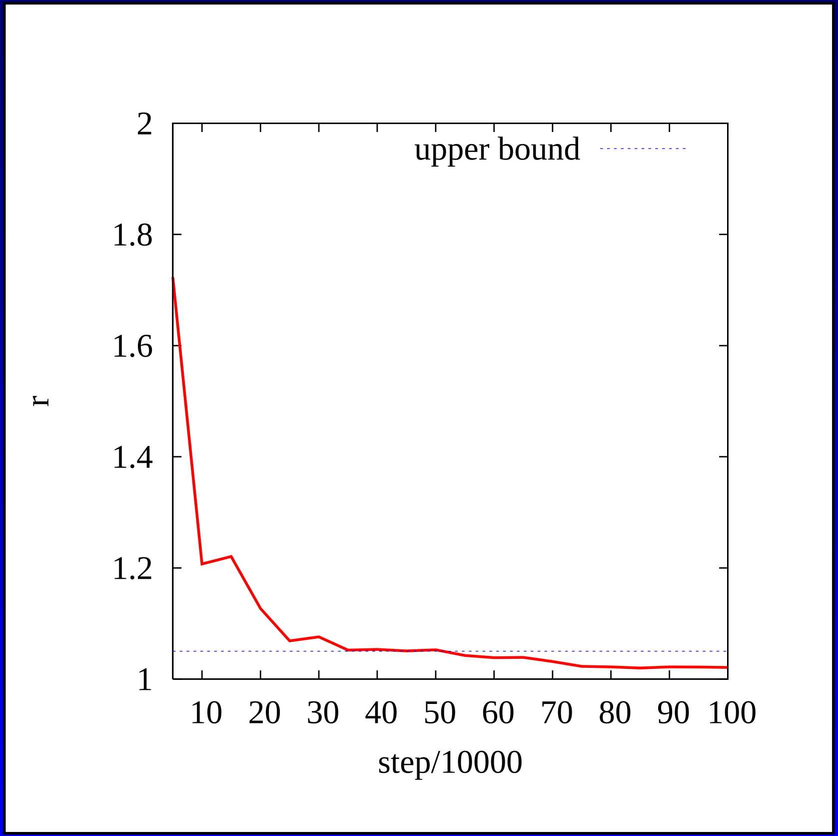
$\mathcal{Q}$: What energy for LC?
$\mathcal{Q}$: What do we get from LHC[a]?

---

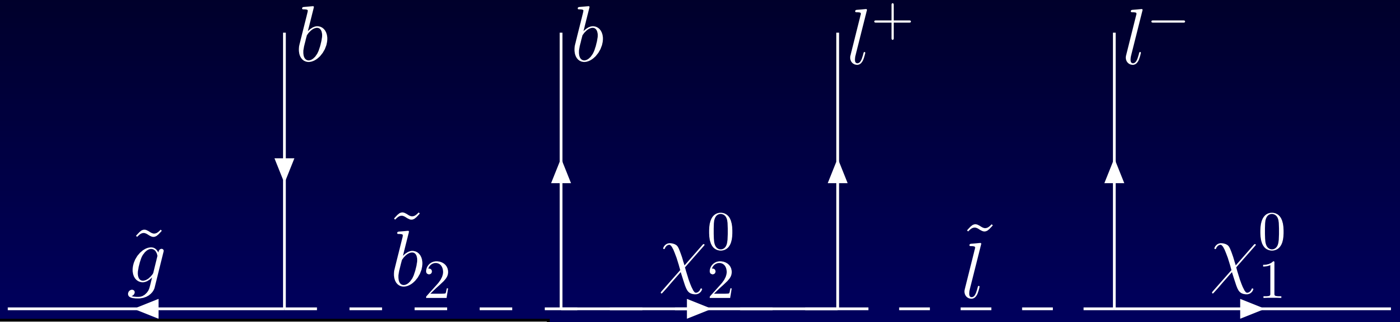
[a] LHC/ILC Working Group Report: `hep-ph/0410364`

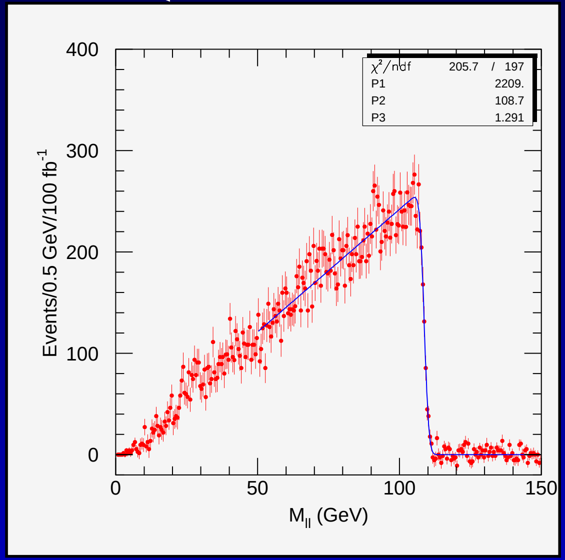
Convergence

We run 9×1 000 000 points. By comparing the 9 independent chains with random starting points, we can provide a statistical measure of convergence: an upper bound r on the excepted variance decrease for infinite statistics.



LHC SUSY Measurements





$$m_{ll}^2 = \frac{(m^2_{\chi^0_2} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\chi^0_1})}{m^2_{\tilde{l}}}$$

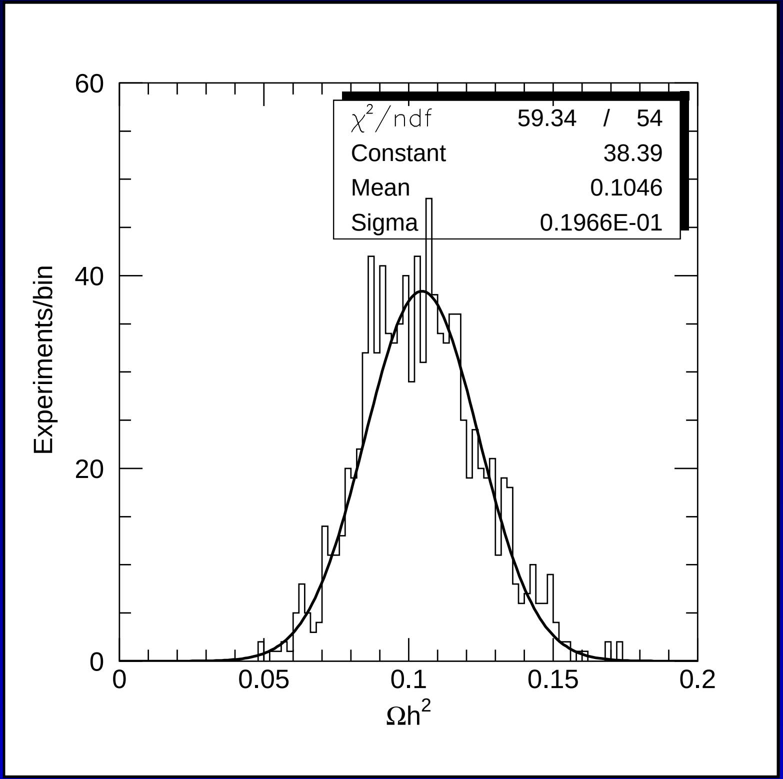
Q: Can we measure enough of these to pin SUSY[a] down?

---

[a] BCA, Lester, Parker, Webber, JHEP 0009 (2000) 004

Predicting Ωh^2

Not much left that's allowed but edge measurements allow reasonable Ωh^2 error[a] for 300 fb^{-1}.



$\mathcal{Q}$: What about other bits of parameter space?

---

[a] M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`.

Bulk Region

M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`. for 300 fb^{-1}. SPA point $m_0 = 70$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -300$ GeV, $\tan\beta = 10$, $\mu > 0$: $\Omega h^2 = 0.108$. Put in m_{ll}^{max}, m_{llq}^{max}, m_{lq}^{low}, m_{lq}^{high}, m_{llq}^{min}, $m_{l_L} - m_{\chi_1^0}$, $m_{ll}^{max}(\chi_4^0)$, $m_{\tau\tau}^{max}$, m_h.

$\tilde\chi_1^0\tilde\chi_1^0 \to \ell^+\ell^-$	40%
$\tilde\chi_1^0\tilde\chi_1^0 \to \tau^+\tau^-$	28%
$\tilde\chi_1^0\tilde\chi_1^0 \to \nu\bar\nu$	3%
$\tilde\chi_1^0\tilde\tau_1 \to Z\tau$	4%
$\tilde\chi_1^0\tilde\tau_1 \to A\tau$	18%
$\tilde\tau_1\tilde\tau_1 \to \tau\tau$	2%

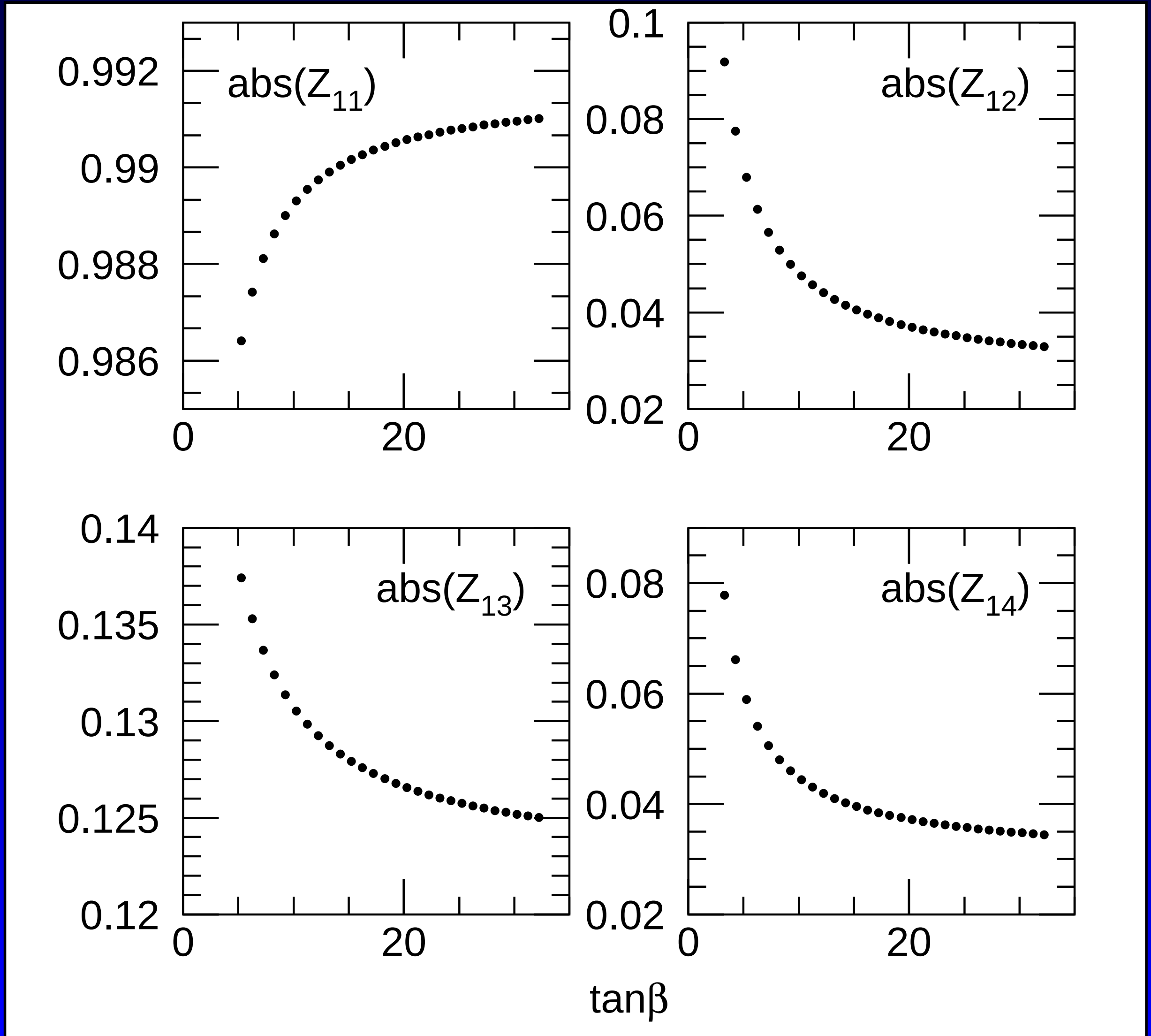
Neutralino mass matrix

Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.

$$\left[\begin{array}{cccc} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{array} \right] \tag{1}$$

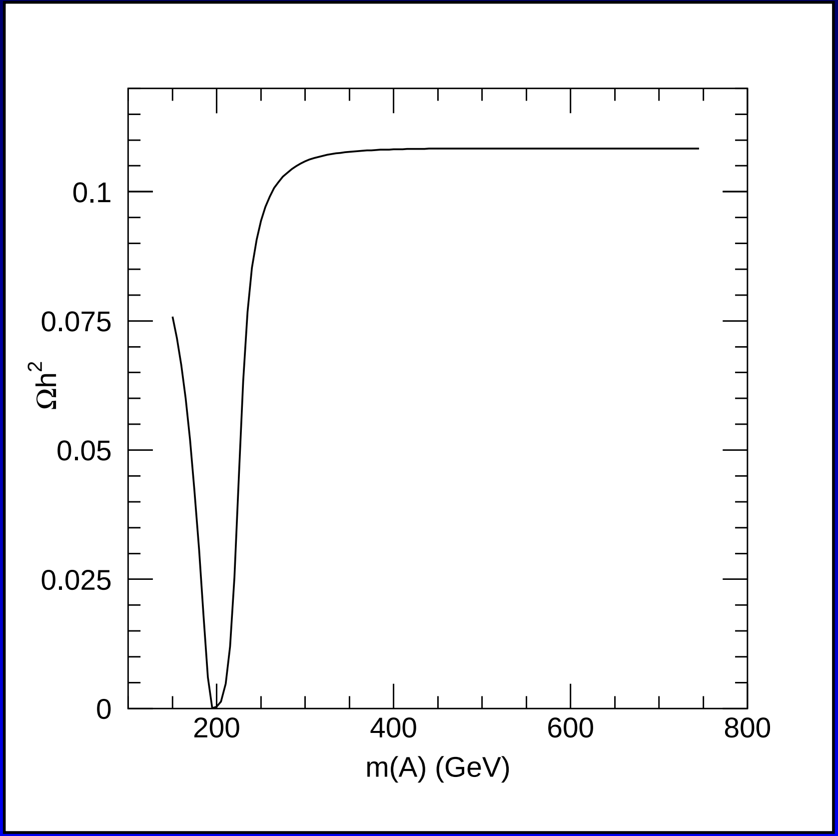
Neutralino mass matrix

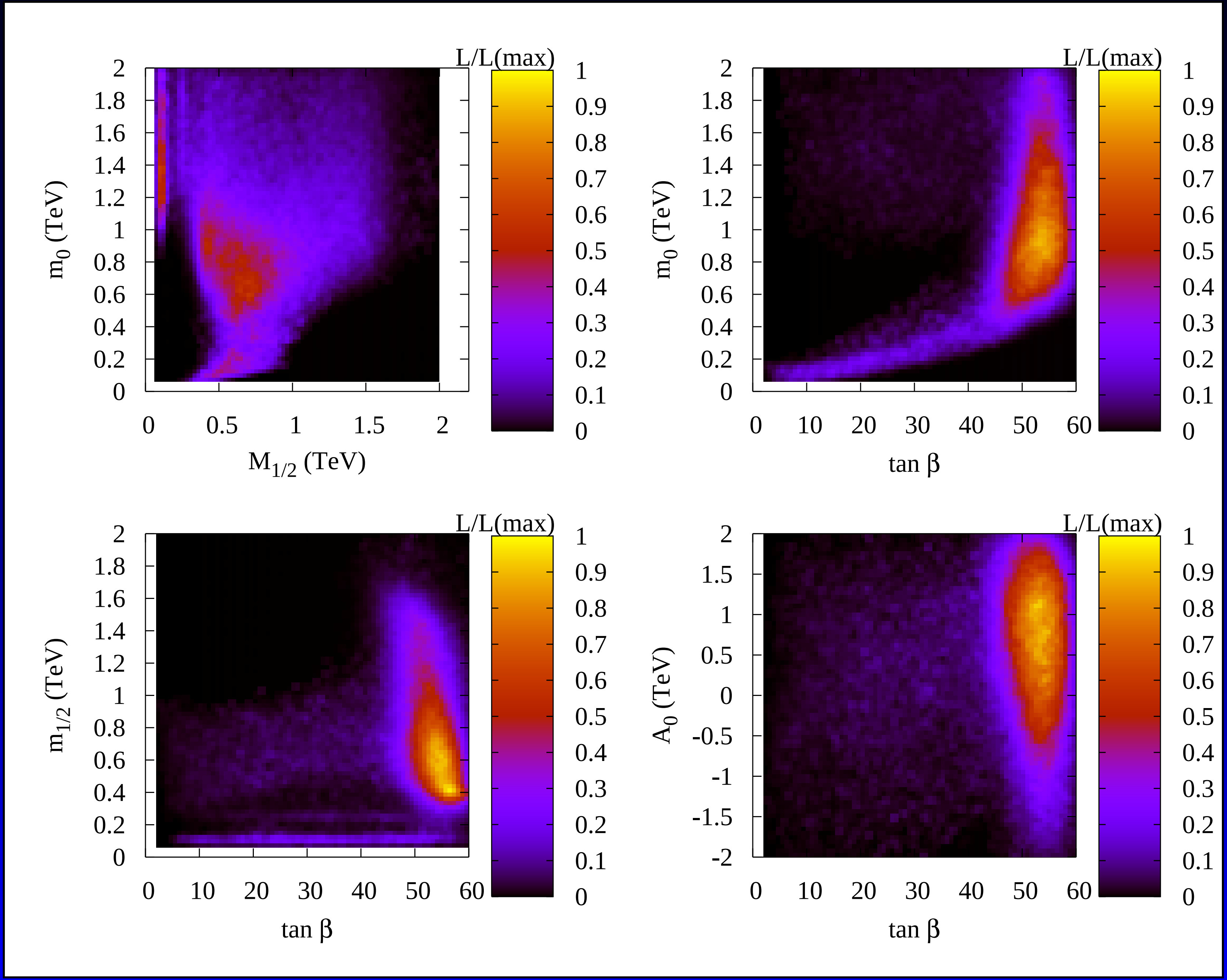
Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.



Slepton/A^0 Higgs

$\Gamma(\chi_2^0 \to \tilde{l}_R l)/\Gamma(\chi_2^0 \to \tilde{\tau}_1 \tau)$ then helps determine θ_τ for a given $\tan\beta$. Exclusion of A^0 helps you to exclude A^0 appearing in cascade decays. Meaurement of m_h provides constraints in $m_A - \tan\beta$ plane.




L/L(max)
m$_0$ (TeV)
M$_{1/2}$ (TeV)
tan β
m$_{1/2}$ (TeV)
A$_0$ (TeV)
0
0.5
1
1.5
2
0.2
0.4
0.6
0.8
1.2
1.4
1.6
1.8
10
20
30
40
50
60
-0.5
-1
-1.5
-2
0.1
0.3
0.5
0.7
0.9

v

Uncertainties in Relic Density

Bulk region: $\tilde{B}\tilde{B} \to Z, h \to l\bar{l}$. Coannihilation: $\tilde{\tau}\chi_1^0 \to \tau + X$

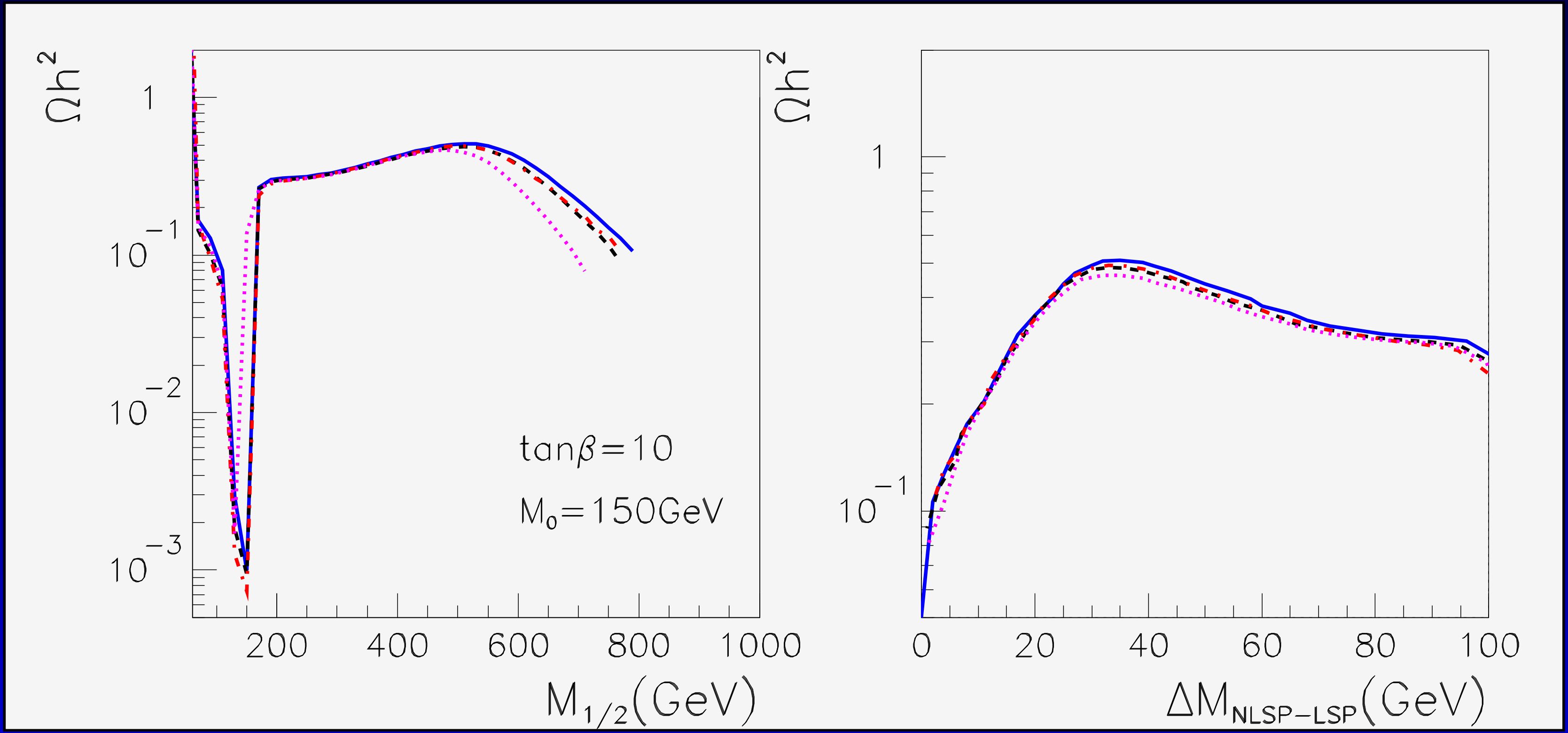


Figure 7: Bulk/coannihilation region. Full: `SoftSusy`, dotted: `SPheno`.

Focus Point

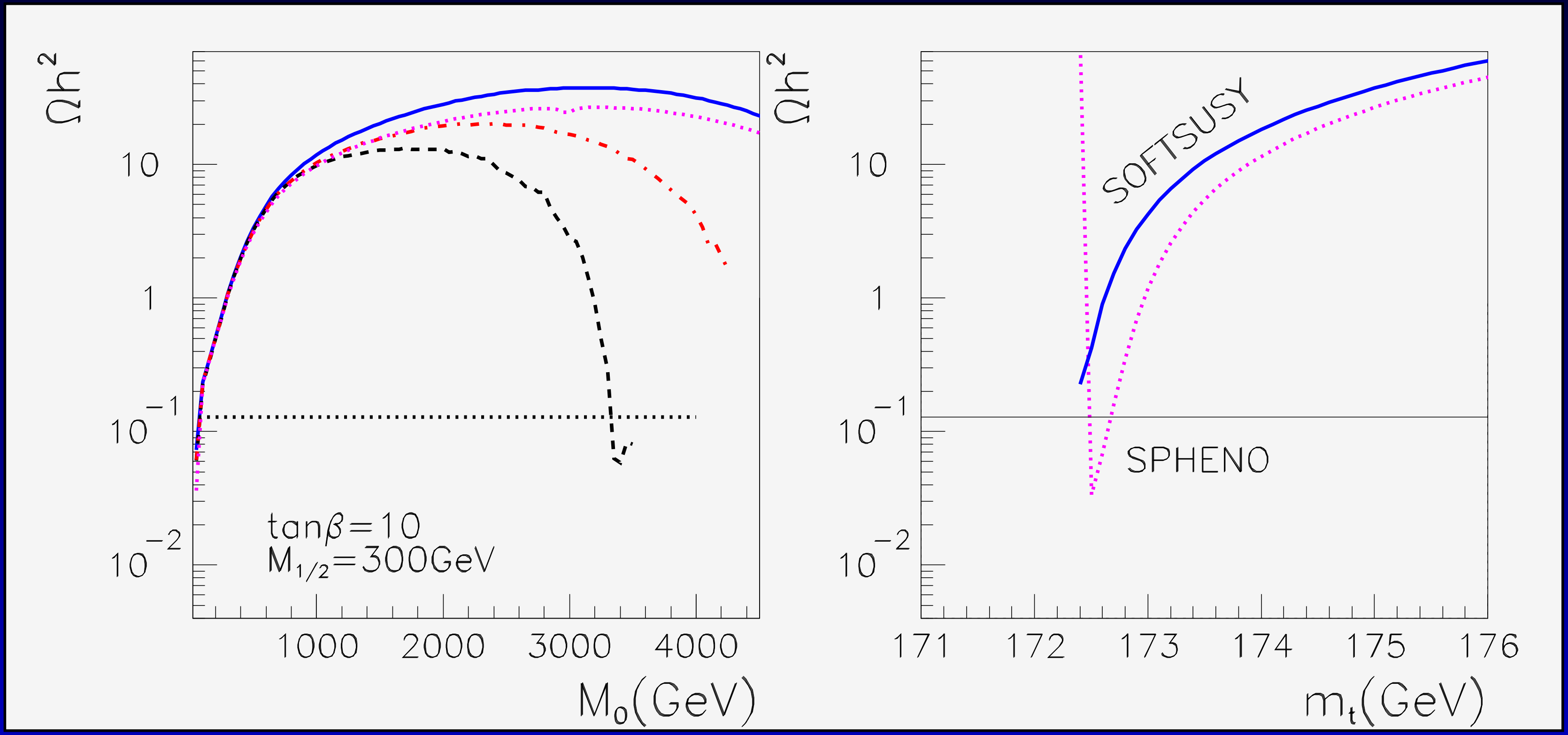


Figure 8: Focus point region. Full: `SoftSusy`, dotted: `SPheno`, dashed: `SuSpect`. Higgsino LSP annihilates into ZZ/WW

High $\tan\beta$

BCA, Belanger, Boudjema, Pukhov, Porod, hep-ph/0402161. Baer *et al*

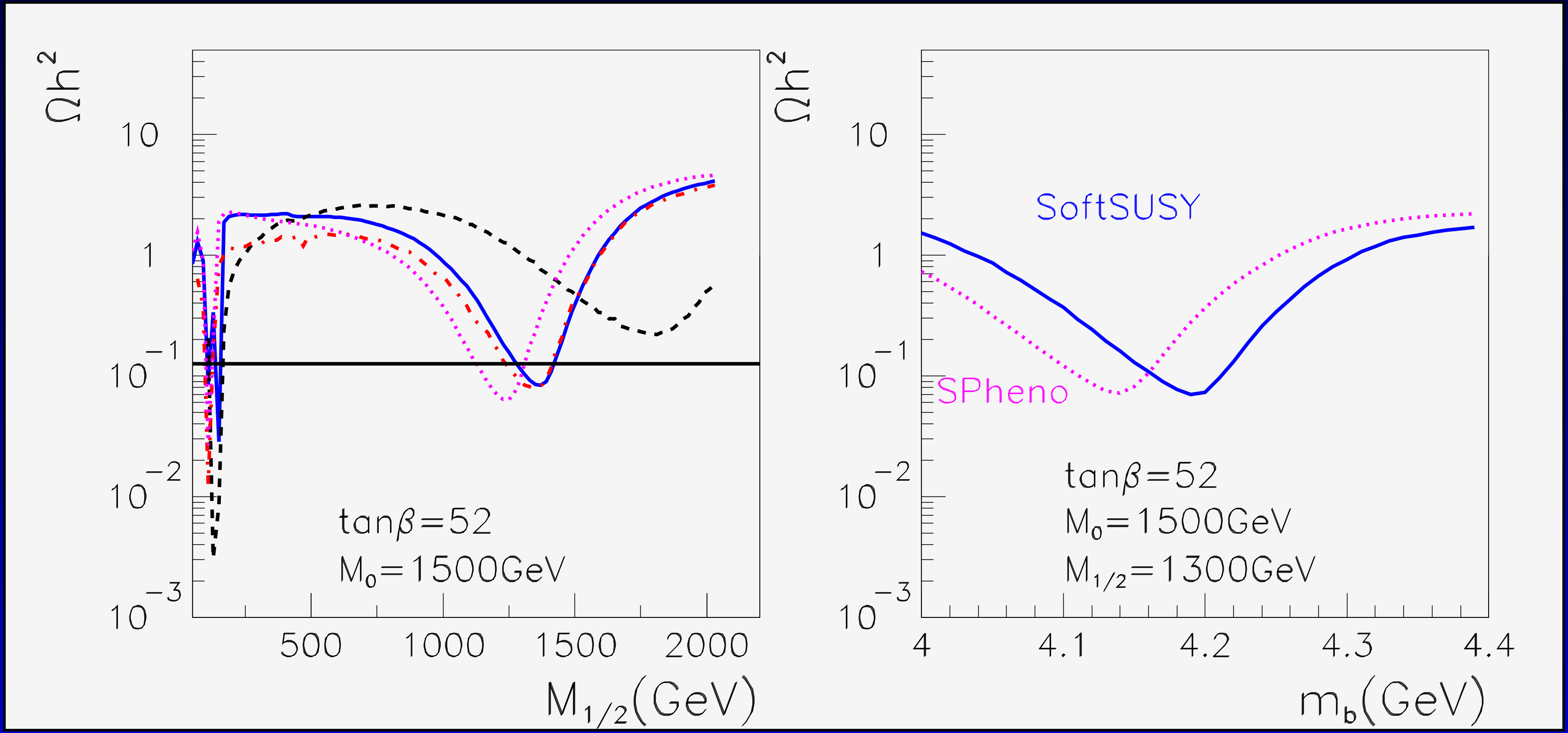
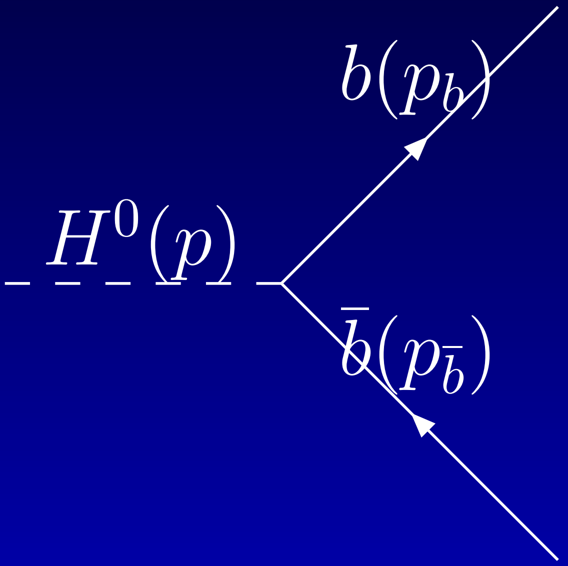


Figure 9: High $\tan\beta$ region. Full: `SoftSusy`, dotted: `SPheno`, dashed: `SuSpect`. Get annihilation into A.

SUSY Kinematics: a Reminder

Take a particle decaying into 2 particles, eg $H^0 \rightarrow b\bar{b}$. We define the invariant mass of the $b\bar{b}$ pair such that:



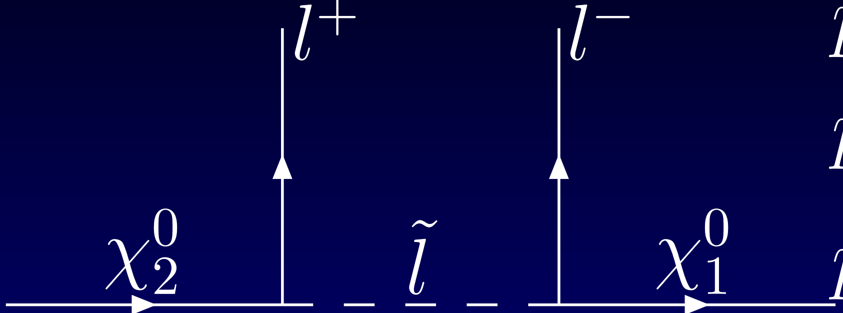
$$p^\mu = (\sqrt{m_H^2 + p^2}, \underline{p}) = p_b^\mu + p_{\bar{b}}^\mu$$

$$\Rightarrow p^2 = m_H^2 = (p_b + p_{\bar{b}})^2$$

Is *invariant* in boosted frames

Question: What happens to invariant mass in SUSY cascade decays, where we miss the final particle?

Cascade Decay



$$p^\mu_{\tilde{l}} = (m_{\tilde{l}}, \underline{0})$$
$$p^\mu_{l^\pm} = (|\underline{p}_{l\pm}|, \underline{p}_{l\pm})$$
$$p^\mu_{\chi^0_{1,2}} = \left(\sqrt{{m_{\chi^0_{1,2}}}^2 + |\underline{p}_{\chi^0_{1,2}}|^2}, \underline{p}_{\chi^0_{1,2}}\right)$$

The invariant mass of the l^+l^- pair is

$$m_{ll}^2 = (p_{l^+} + p_{l^-})^\mu (p_{l^+} + p_{l^-})_\mu = p_{l^+}^2 + p_{l^-}^2 + 2 p_{l^+} \cdot p_{l^-}$$
$$= 2|\underline{p}_{l+}||\underline{p}_{l-}|(1 - \cos\theta) \leq 4|\underline{p}_{l+}||\underline{p}_{l-}|.$$

Momentum conservation:

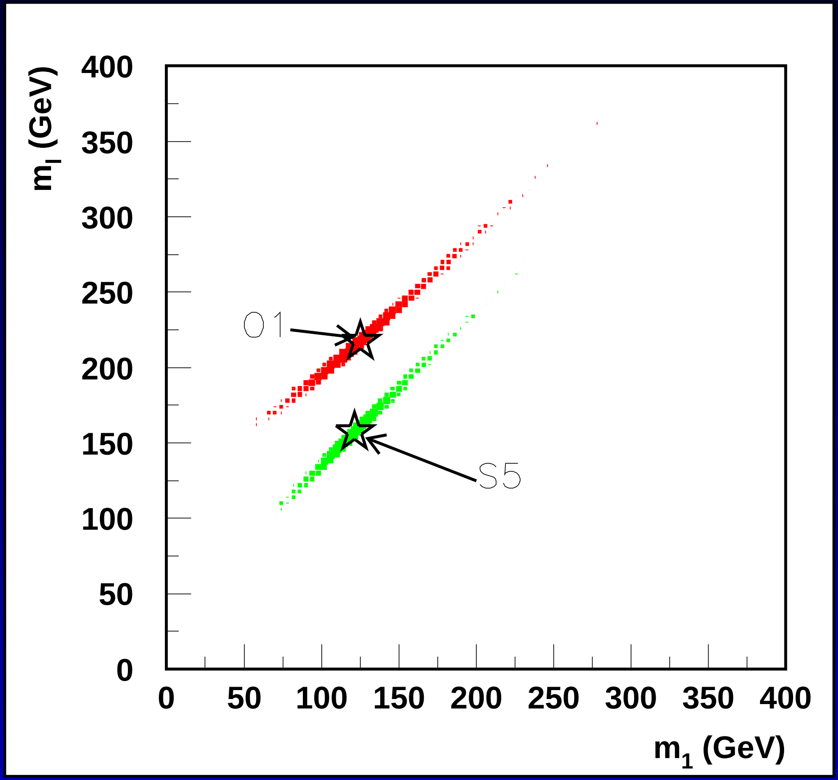
$$\Rightarrow \underline{p}_{\chi_2^0} + \underline{p}_{l+} = \underline{0}, \qquad \underline{p}_{l-} + \underline{p}_{\chi_1^0} = \underline{0}.$$

Energy conservation: $\sqrt{{m_{\chi_2^0}}^2 + |\underline{p}_{l+}|^2} = m_{\tilde{l}} + |\underline{p}_{l+}|$,

$$\Rightarrow |\underline{p}_{l+}| = \frac{m^2_{\chi^0_2} - m^2_{\tilde{l}}}{2m_{\tilde{l}}}.\ \text{Similarly } |\underline{p}_{l-}| = \frac{m^2_{\tilde{l}} - m^2_{\chi^0_1}}{2m_{\tilde{l}}}.$$

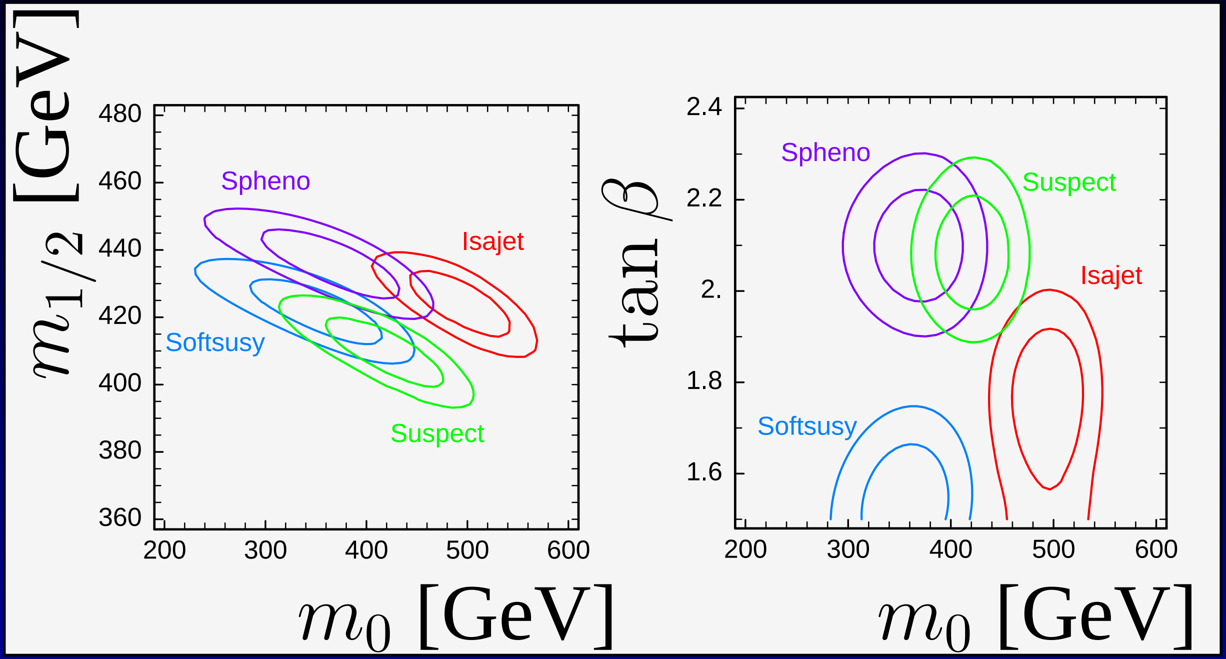
Edge to Mass Measurements

	width S5	width O1
χ_1^0	17	22
$\tilde{l}_R$	17	20
χ_2^0	17	20
$\tilde{q}$	22	20



Mass differences well constrained, but overall mass scale not so well constrained by LHC

Fitting to SUSY Breaking Model



- Experimenters pick a SUSY breaking point
- They derive observables and errors after detector simulation
- We fit[a] this "data" with our codes

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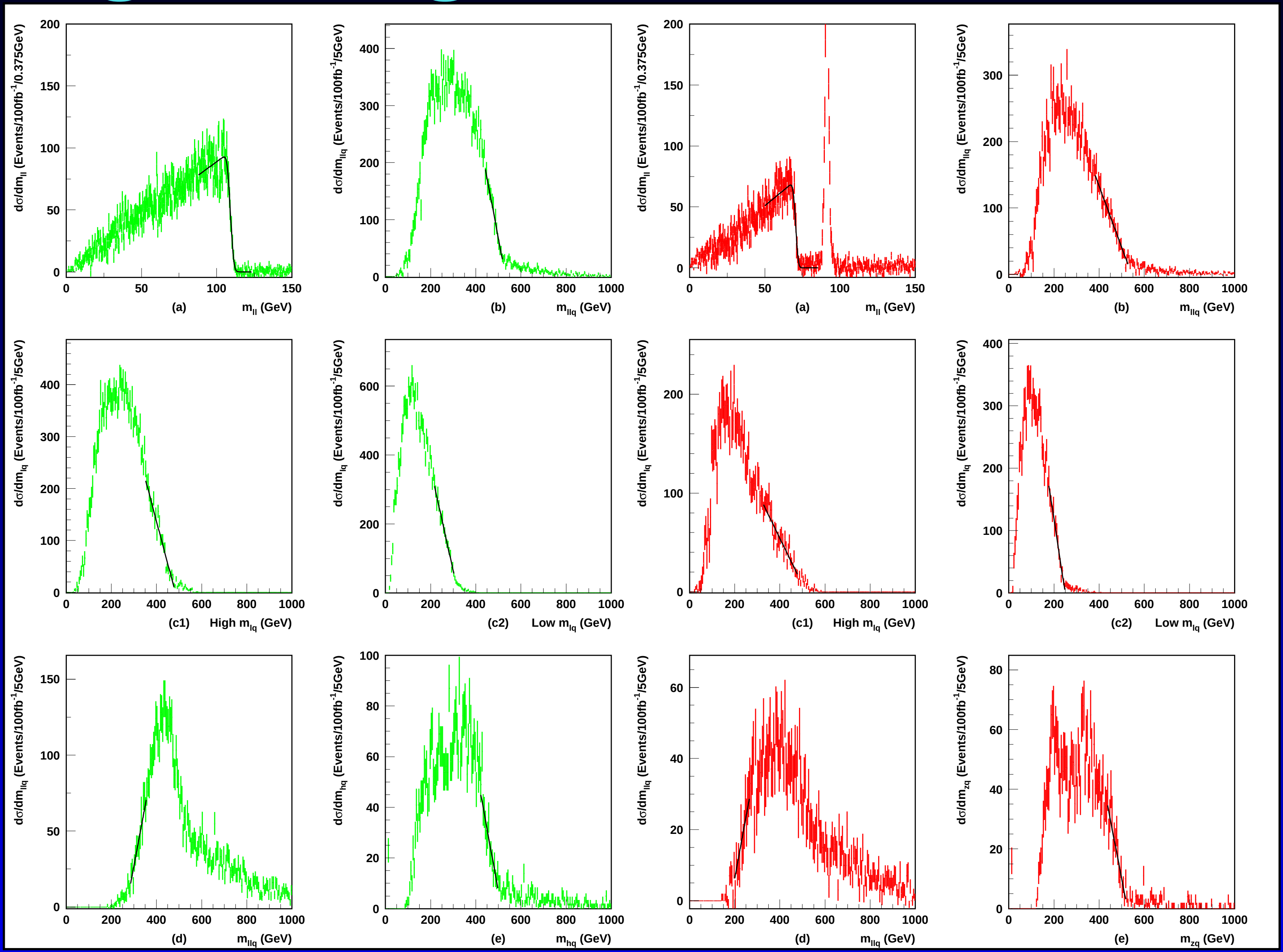
[a] BCA, S Kraml, W Porod, JHEP 0303 (2003) 016

Edge Fitting at S5 and O1

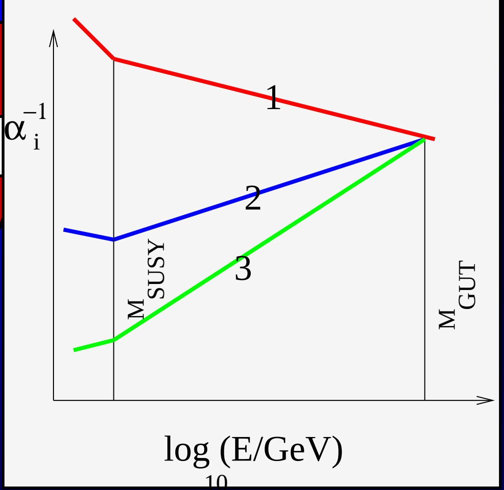


Edge Positions

endpoint	S5 fit	O1 fit
m_{ll}	109.10 ± 0.13	70.47±0.15
m_{llq} edge	532.1 ± 3.2	544.1±4.0
lq high	483.5 ± 1.8	515.8±7.0
lq low	321.5 ± 2.3	249.8±1.5
llq thresh	266.0 ± 6.4	182.2±13.5

Best case lepton mass measurements can be as accurate as 1 per mille, but jets are a few percent

SOFTSUSY



Get $g_i(M_Z)$, $h_{t,b,\tau}(M_Z)$.

↓

Run to M_S.

↓

REWSB, iterative solution of μ

↓

M_X. Soft SUSY breaking BC.

↓

Run to M_S. Calculate[a] sparticle pole masses.

↓

Run to M_Z

---

[a] BCA, Comp. Phys. Comm. 143 (2002) 305.

Other Observables

Often more complicated, eg m_{llq} edge:

$$\max\left[\frac{(m_{\tilde q}^2 - m_{\chi_2^0}^2)(m_{\chi_2^0}^2 - m_{\chi_1^0}^2)}{m_{\chi_2^0}^2}, \frac{(m_{\tilde q}^2 - m_{\tilde l}^2)(m_{\tilde l}^2 - m_{\chi_1^0}^2)}{m_{\tilde l}^2}, \frac{(m_{\tilde q} m_{\tilde l} - m_{\chi_2^0} m_{\chi_1^0})(m_{\chi_2^0}^2 - m_{\tilde l}^2)}{m_{\chi_2^0} m_{\tilde l}}\right]$$

Also m_{lq}^{high}, m_{lq}^{low}, llq *threshold* [a], $M_{T_2}^2(m) =$

$$\min_{\not p_1 + \not p_2 = \not p_T} \left[\max\left\{ m_T^2(p_T^{l_1}, \not p_1, m), m_T^2(p_T^{l_2}, \not p_2, m) \right\}\right],$$

$\max[M_{T_2}(m_{\chi_1^0})] = m_{\tilde l}]$ for dislepton production.

Same order prior

We wish to encode the idea that "SUSY breaking terms should be of the same order of magnitude"

$$p(m_0|M_S) = \frac{1}{\sqrt{2\pi w^2 m_0}} \exp\left(-\frac{1}{2w^2}\log^2(\frac{m_0}{M_S})\right),$$

$$p(A_0|M_S) = \frac{1}{\sqrt{2\pi e^{2w} M_S}} \exp\left(-\frac{1}{2e^{2w}}\frac{A_0^2}{M_S^2}\right),$$

We don't know SUSY breaking scale M_S:

$$p(m_0, M_{1/2}, A_0, \mu, B) = \int_0^\infty dM_S\, p(m_0, M_{1/2}, A_0, \mu, B|M_S)\, p(M_S)$$

Naturalness

$$M_Z^2 = \tan 2\beta \left[m_{H_2}^2 \tan\beta - m_{H_1}^2 \cot\beta \right] - 2\mu^2$$

Cancellation implied by sparticle mass bounds. Quantify by

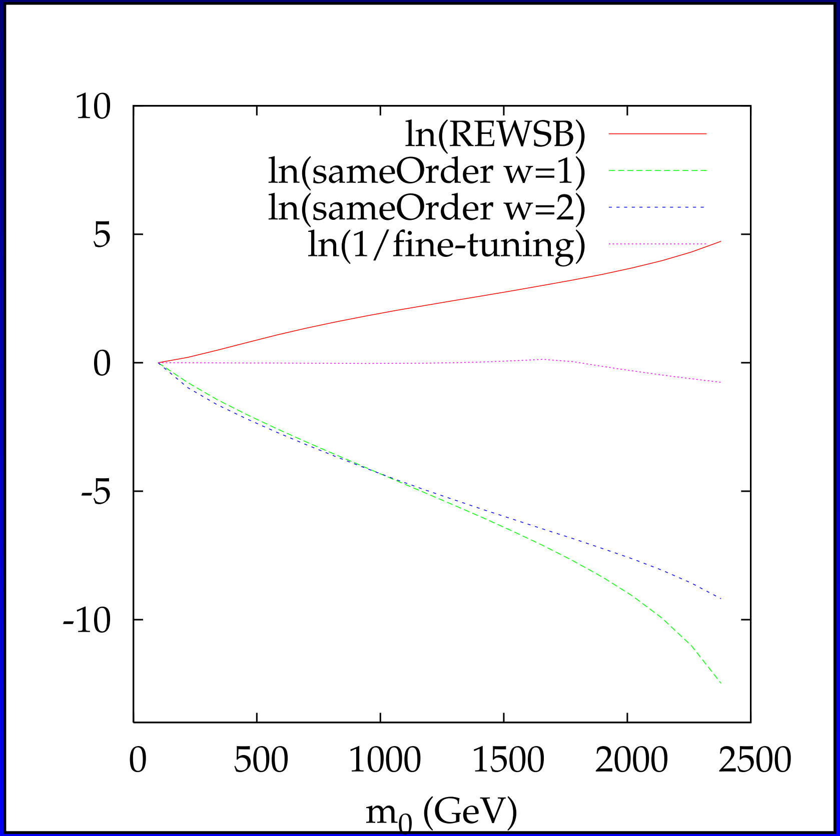
$$f = \max_x \{ \| \frac{d \ln M_Z^2}{d \ln x} \| \}$$

where $x \in \{M_{1/2}, m_0, A_0, \mu, B\}$. We will choose the prior to be $1/f$.

Fine Tuning

Compare with usual definition of *fine-tuning*:

$$f = \max_p \frac{d \ln M_Z}{d \ln p}$$



SPS1a Point

$M_{1/2} = 250$ GeV

$\tan\beta = 10$ GeV

$A_0 = -100$ GeV