Fermilab U.S. DEPARTMENT OF ENERGY | Office of Science

Aspects of neutrino masses

Jessica Turner

UCL

13 December 2019

Outline

- Neutrino masses and mixing
- Consequences of neutrino masses
- Neutrino masses from gravity, what has been done
- Neutrino masses from gravity: the Schwinger Dyson approach
- Two ways of solving the SDEs
- Discussions and conclusions

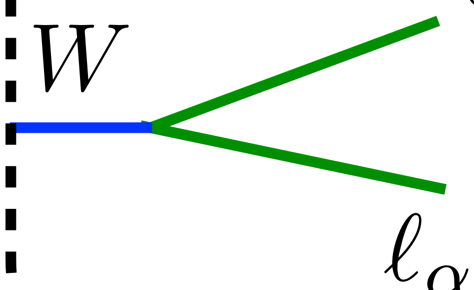
Neutrino Oscillations

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

flavour states — **PMNS matrix** — **mass states**

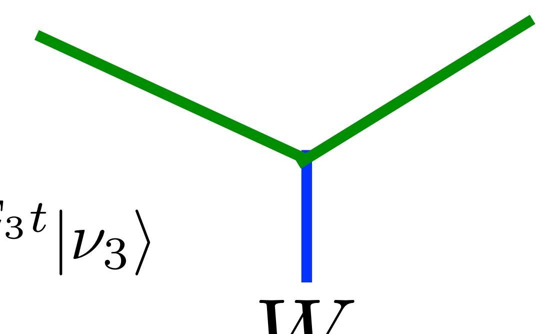
$$U m_\nu U^\dagger = m_{\nu \text{diag}}$$

$$U_{PMNS} = U_e^\dagger U_\nu$$



$\nu_\alpha(t_0) \longrightarrow \nu_\beta(t)$

$$|\nu(t)\rangle = e^{-i\mathcal{H}t}|\nu(0)\rangle = \alpha e^{-iE_1 t}|\nu_1\rangle + \beta e^{-iE_2 t}|\nu_2\rangle + \gamma e^{-iE_3 t}|\nu_3\rangle$$



$$E_i \simeq E + \frac{m_i^2}{E} \implies E_i - E_j \simeq \frac{\Delta m_{ij}^2}{2E}$$

In the simplified two neutrino case:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2\frac{(\Delta m_{ij}^2 L)}{2E}$$

mass splitting

baseline distance between production and detection

mixing angle parametrises misalignment of bases

Energy of neutrino

- Neutrinos have (non-degenerate) masses
- Neutrinos mix i.e. PMNS matrix is a non-identity matrix

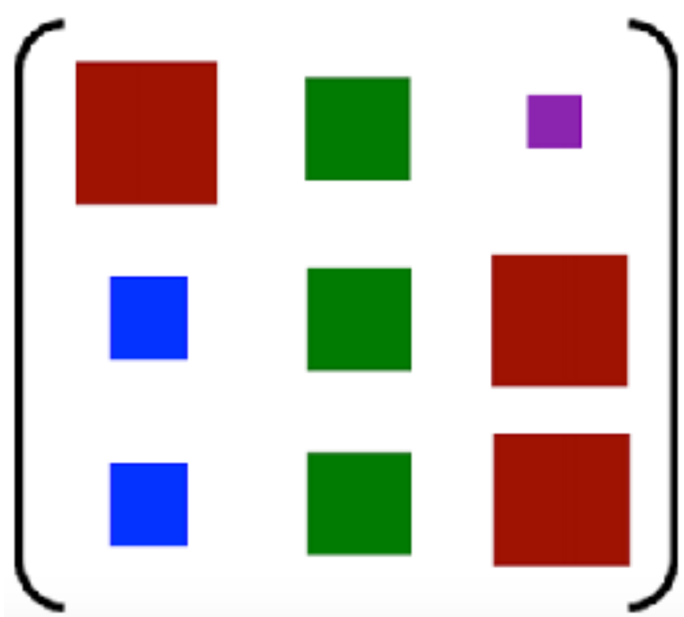
- If neutrinos Dirac fermions, PMNS: 3 mixing angles + 1 phase

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

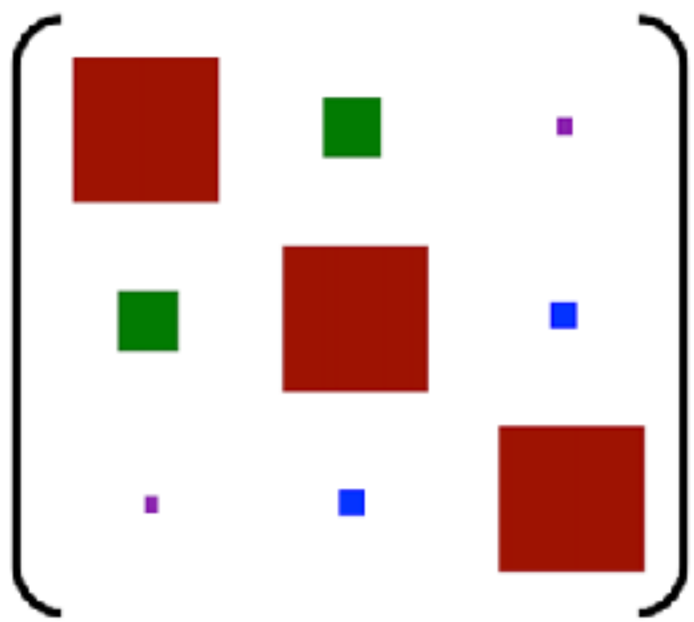
$$41.1 \leq \theta_{23}(^\circ) \leq 51.3 \qquad 8.22 \leq \theta_{13}(^\circ) \leq 8.98 \qquad 31.61 \leq \theta_{12}(^\circ) \leq 36.27$$

$$144 \leq \delta(^\circ) \leq 357$$

nu-fit data 4.1



leptonic mixing



quark mixing

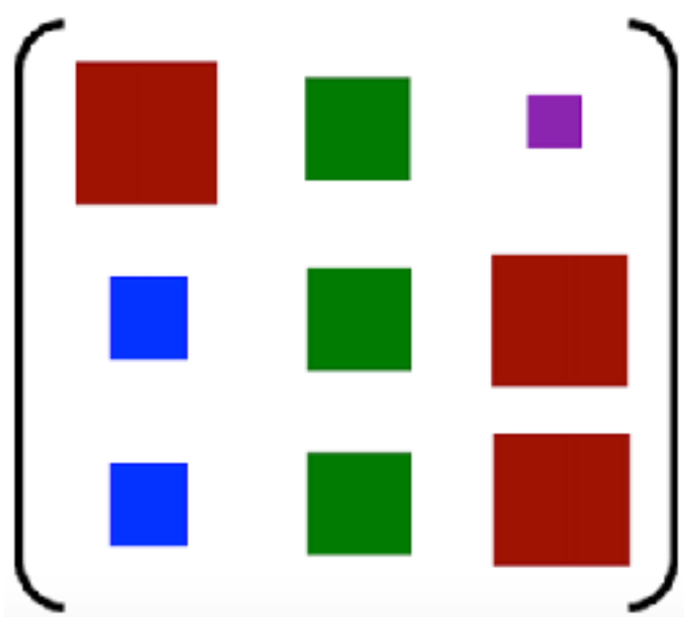
- Neutrinos have (non-degenerate) masses
- Neutrinos mix i.e. PMNS matrix is a non-identity matrix

- If neutrinos Majorana fermions, PMNS: 3 mixing angles + 3 phase

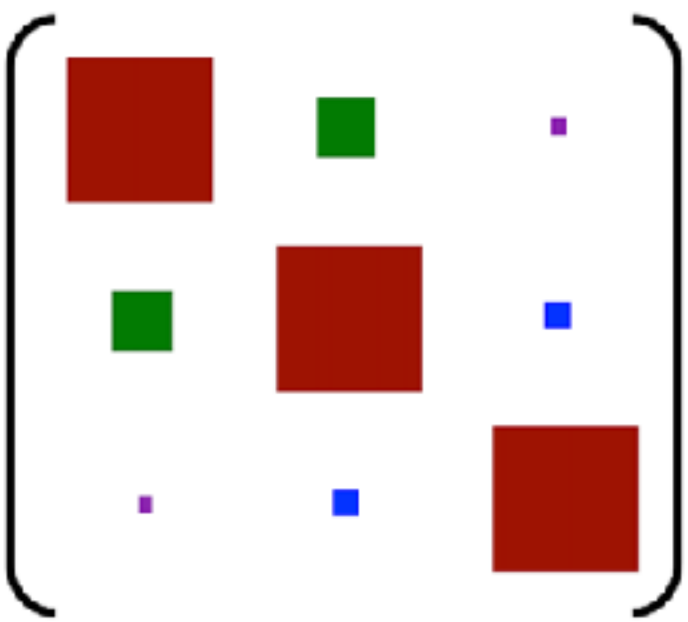
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

- Majorana nature of neutrinos not observable at oscillation experiments

nu-fit data 4.1

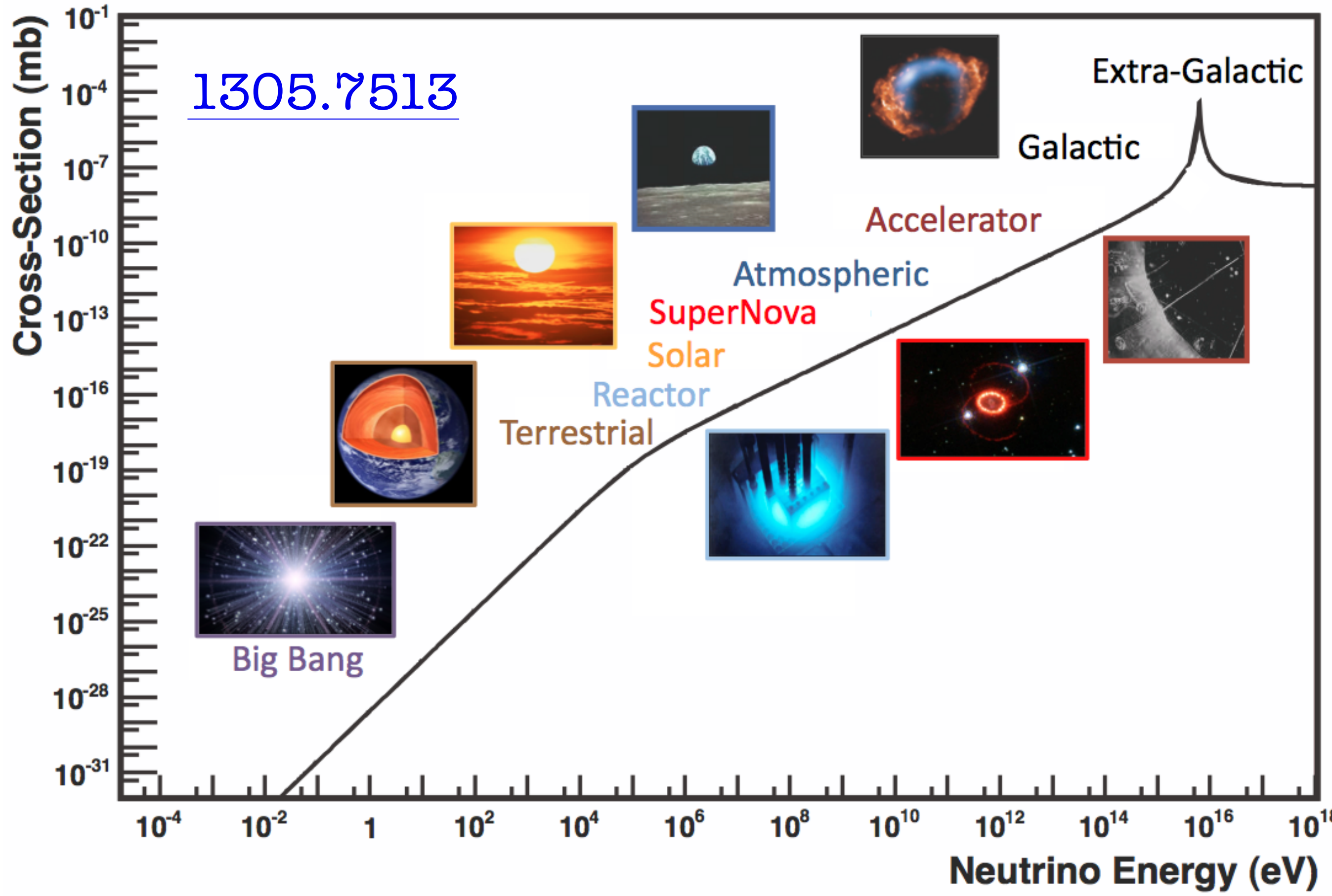


leptonic mixing



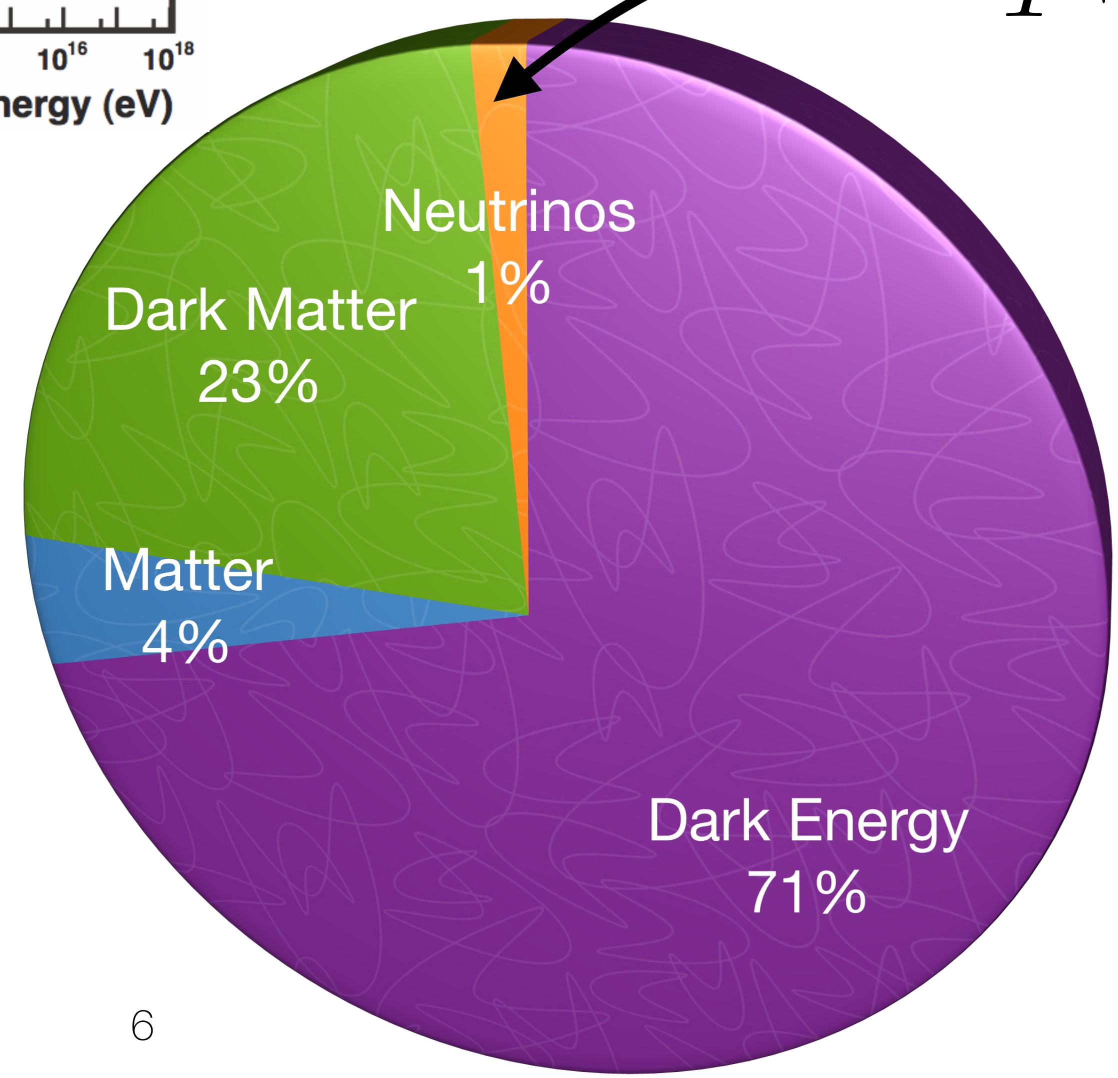
quark mixing

Neutrinos are ubiquitous but also mysterious



$\sim 330\,\mathrm{cm}^{-3}$

$T \sim 1.95\,\mathrm{K}$

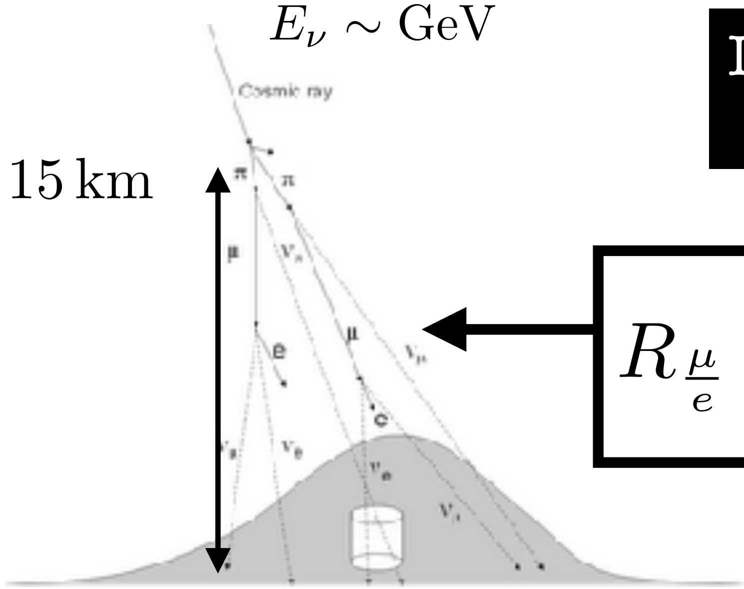


Atmospheric

$\nu_{e,\mu}$ produced by interaction of cosmic rays with Earth's atmosphere

$E_\nu \sim \mathrm{GeV}$

15 km



Discovered by SK in 1998

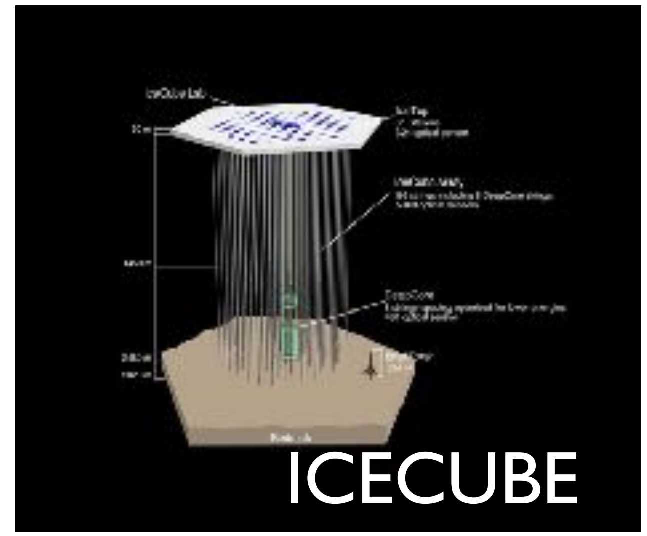
$$R_{\frac{\mu}{e}} = \frac{N_{\nu_\mu} + N_{\overline{\nu_\mu}}}{N_{\nu_e} + N_{\overline{\nu_e}}} \approx 2$$

$$P\left(\nu_\mu \rightarrow \nu_\tau\right)$$

$$\Delta m^2 \sim 2 \times 10^{-3}\,\mathrm{eV}^2$$

$$\theta \sim \frac{\pi}{4}$$

$E_\nu \sim \mathrm{GeV}$



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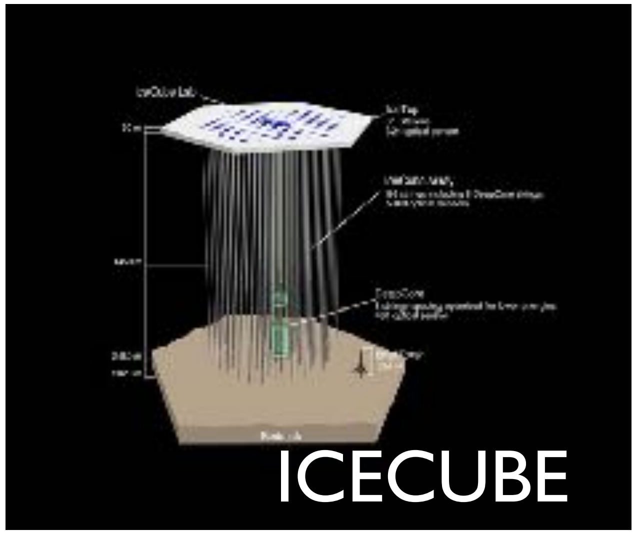
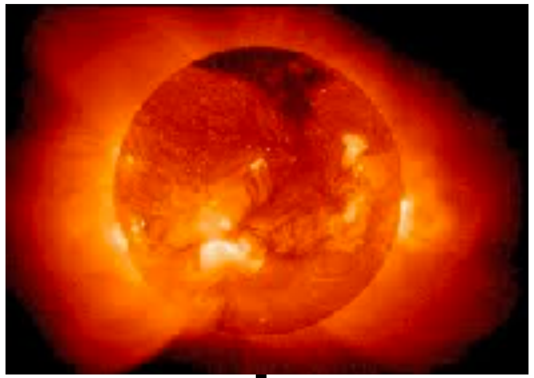


Solar



dominant energy production

CNO cycle

pp chain

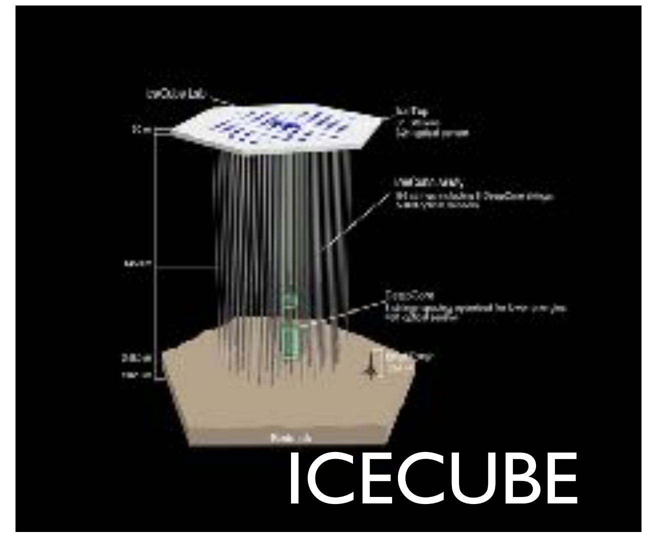
ν_e copiously produced by the Sun

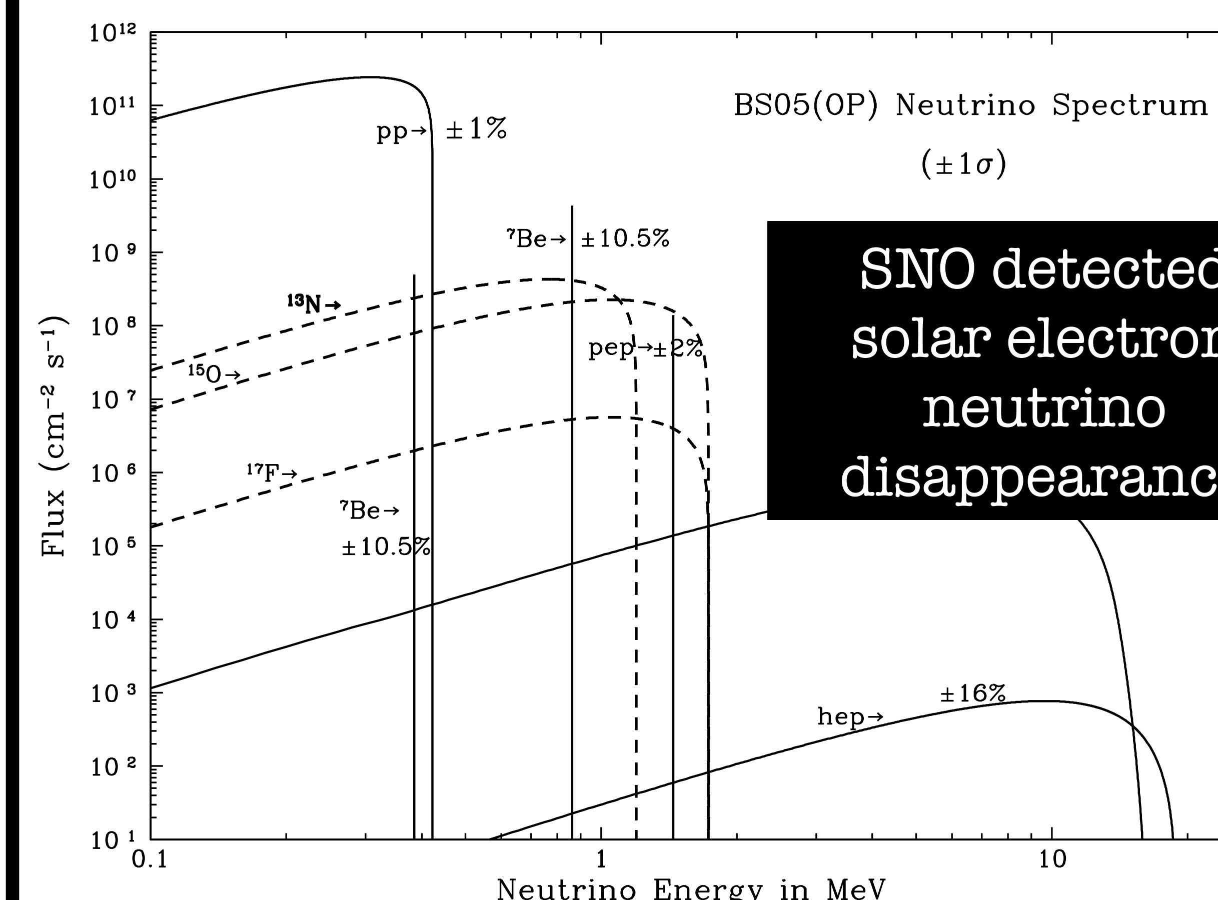
$$P\left(\nu_\mu \rightarrow \nu_\tau\right)$$

$$\Delta m^2 \sim 2 \times 10^{-3}\,\mathrm{eV}^2$$

$$\theta \sim \frac{\pi}{4}$$

$$E_\nu \sim \mathrm{GeV}$$





SNO detected solar electron neutrino disappearance

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$$P\left(\nu_\mu \to \nu_\tau\right)$$

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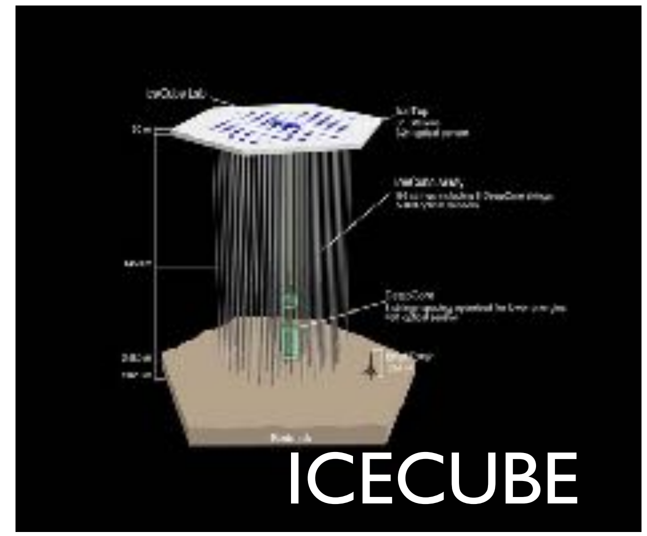
$$\theta \sim \frac{\pi}{4}$$

$E_\nu \sim \mathrm{GeV}$



$$P\left(\nu_e \to \nu_{\mu/\tau}\right)$$

$$\Delta m^2 \sim 7 \times 10^{-5}\,\mathrm{eV}^2$$

terrestrial source

$$\theta \sim \frac{\pi}{6}$$

$L \sim 10^8\,\mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$L \sim 180\,\mathrm{km}$

$$P\left(\nu_\mu \to \nu_\tau\right)$$

$$\Delta m^2 \sim 2 \times 10^{-3}\,\mathrm{eV}^2$$

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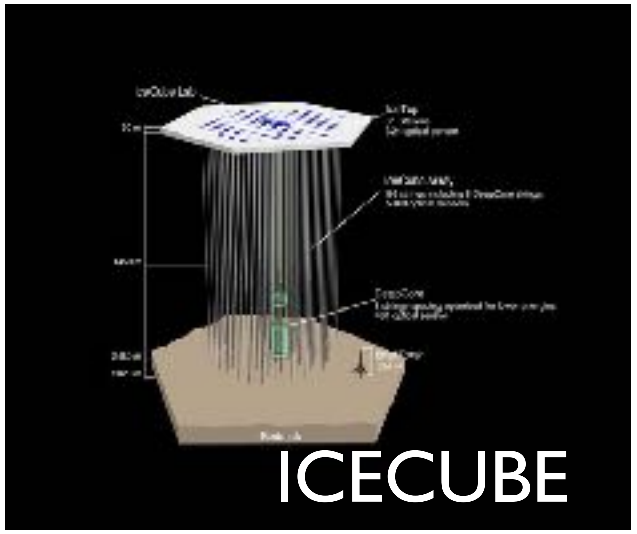
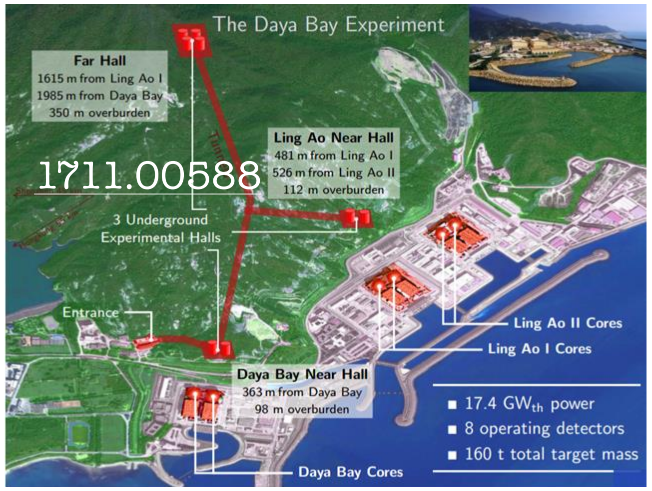


Reactor

DAYA, RENO and Double Chooz all use neutrinos produced at reactors



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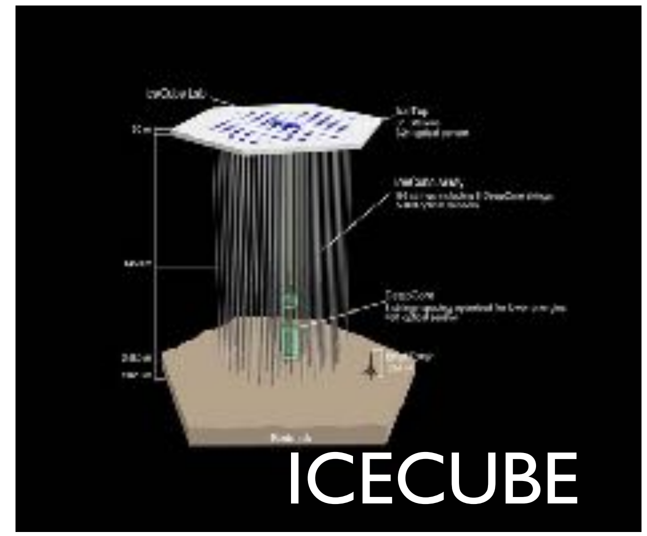


$$P\left(\overline{\nu}_e \to \overline{\nu}_e\right)$$

$$\theta \sim \frac{\pi}{20}$$

$L \sim \mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$$P\left(\nu_e \to \nu_{\mu/\tau}\right)$$

$$\Delta m^2 \sim 7 \times 10^{-5}\,\mathrm{eV}^2$$

$$\theta \sim \frac{\pi}{6}$$

terrestrial source

$L \sim 10^8\,\mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$L \sim 180\,\mathrm{km}$

Accelerator

NOVA $L \sim 810\,\mathrm{km}$

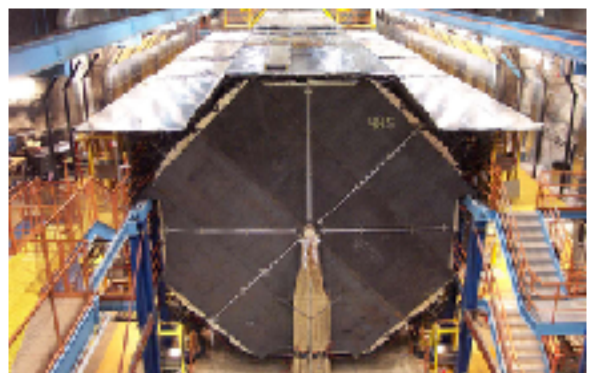
$E_\nu \sim \mathcal{O}(1)\,\mathrm{GeV}$

T2K

$L \sim 295\,\mathrm{km}$

$E_\nu \sim \mathcal{O}(100)\,\mathrm{MeV}$

MINOS $L \sim 735\,\mathrm{km}$

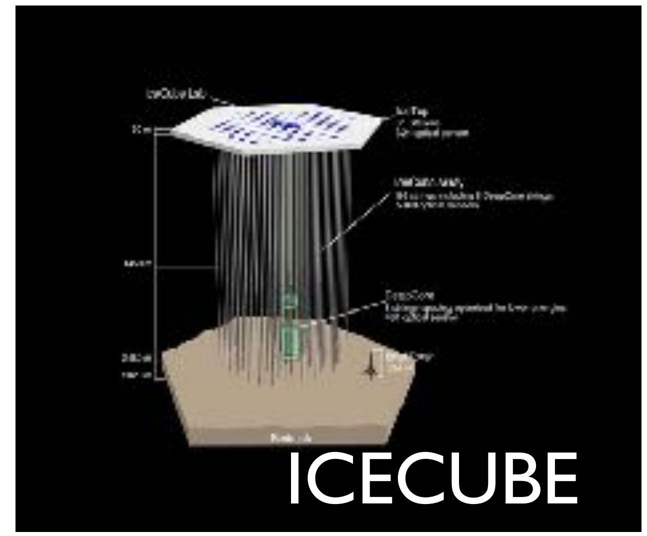


$P\left(\nu_\mu \to \nu_\tau\right)$

$\Delta m^2 \sim 2 \times 10^{-3}\,\mathrm{eV}^2$

$\theta \sim \dfrac{\pi}{4}$

$E_\nu \sim \mathrm{GeV}$



$P\left(\overline{\nu}_e \to \overline{\nu}_e\right)$

$\theta \sim \dfrac{\pi}{20}$

$L \sim \mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$P\left(\nu_e \to \nu_{\mu/\tau}\right)$

$\Delta m^2 \sim 7 \times 10^{-5}\,\mathrm{eV}^2$

terrestrial source

$\theta \sim \dfrac{\pi}{6}$

$L \sim 10^8\,\mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$L \sim 180\,\mathrm{km}$

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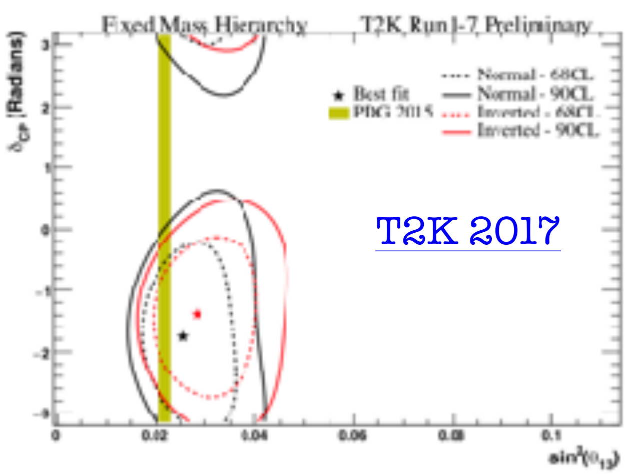
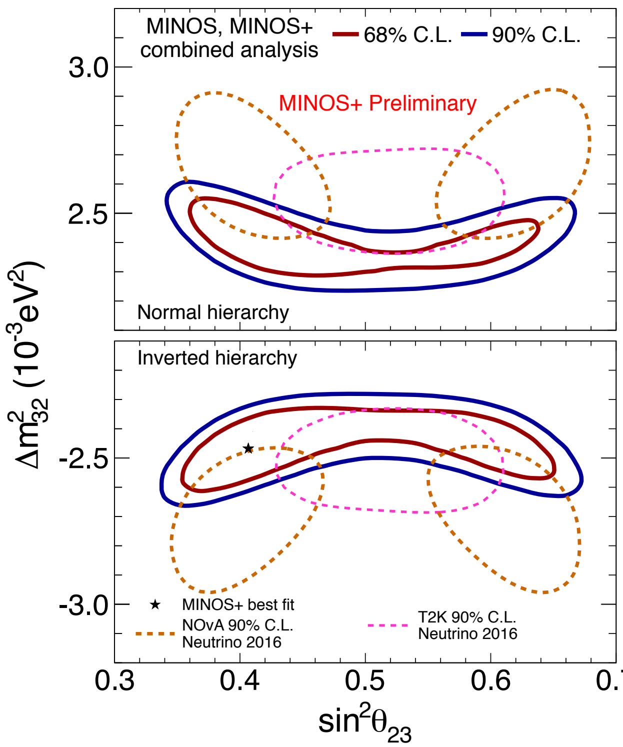
Accelerator

$\Delta m^2 \sim 10^{-3}\,\mathrm{eV}^2$

$P(\nu_\mu \to \nu_{\alpha\neq\mu})$ $\quad \theta_{23}$

$$\delta \sim \frac{3\pi}{2}$$



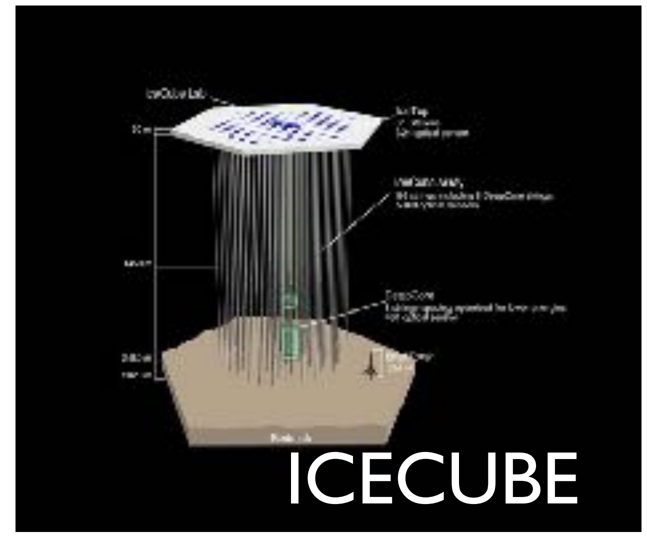


$P(\nu_\mu \to \nu_\tau)$

$\Delta m^2 \sim 2 \times 10^{-3}\,\mathrm{eV}^2$

$E_\nu \sim \mathrm{GeV}$

$$\theta_{23} \sim \frac{\pi}{4}$$



$P(\overline{\nu}_e \to \overline{\nu}_e)$

$L \sim \mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$$\theta_{13} \sim \frac{\pi}{20}$$

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$P(\nu_e \to \nu_{\mu/\tau})$

$\Delta m^2 \sim 7 \times 10^{-5}\,\mathrm{eV}^2$

terrestrial source

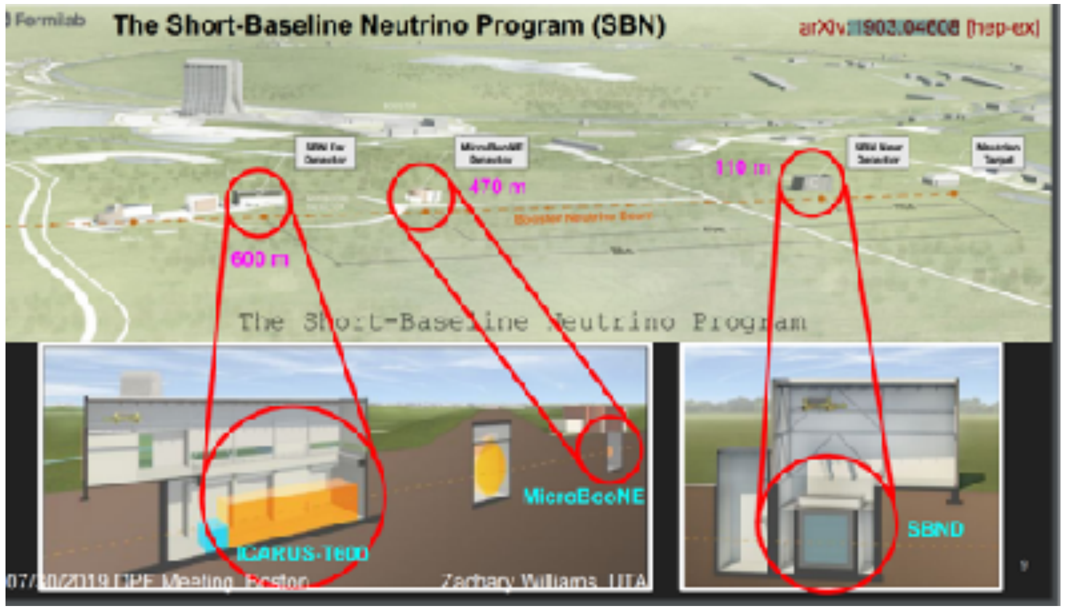
$$\theta_{12} \sim \frac{\pi}{6}$$

$L \sim 10^8\,\mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

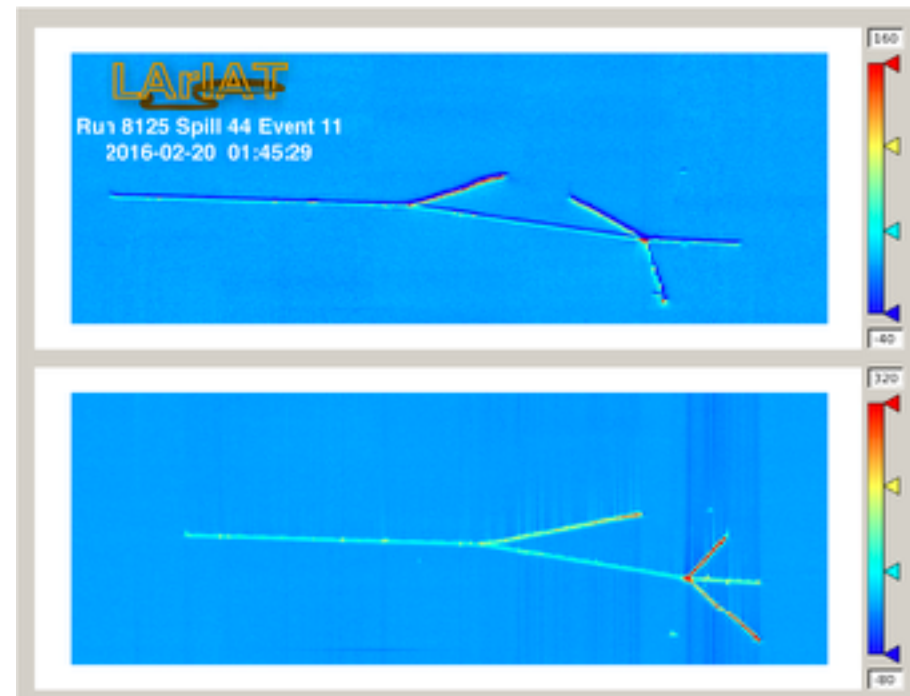
$L \sim 180\,\mathrm{km}$

Other experiments and future ones

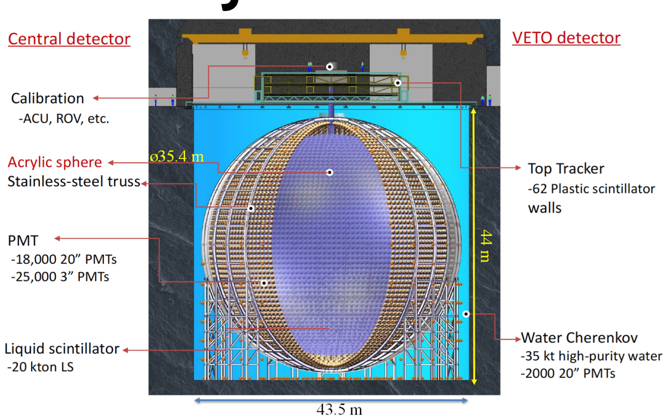


Liquid argon detectors, beam ~ O(1) GeV

- investigate anomalies eV scale steriles
- neutrino argon cross sections

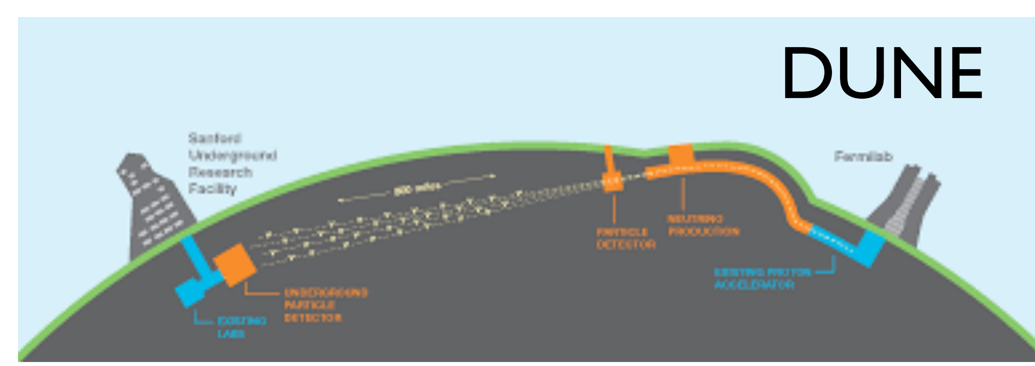


JUNO



Mass ordering and solar mixing

CPV, atmospheric mixing, proton decay, mass ordering….



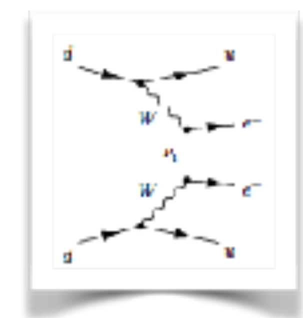
CPV, atmospheric mixing, proton decay, mass ordering….

Majorana neutrinos

If neutrinos are Majorana there are two additional phases:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

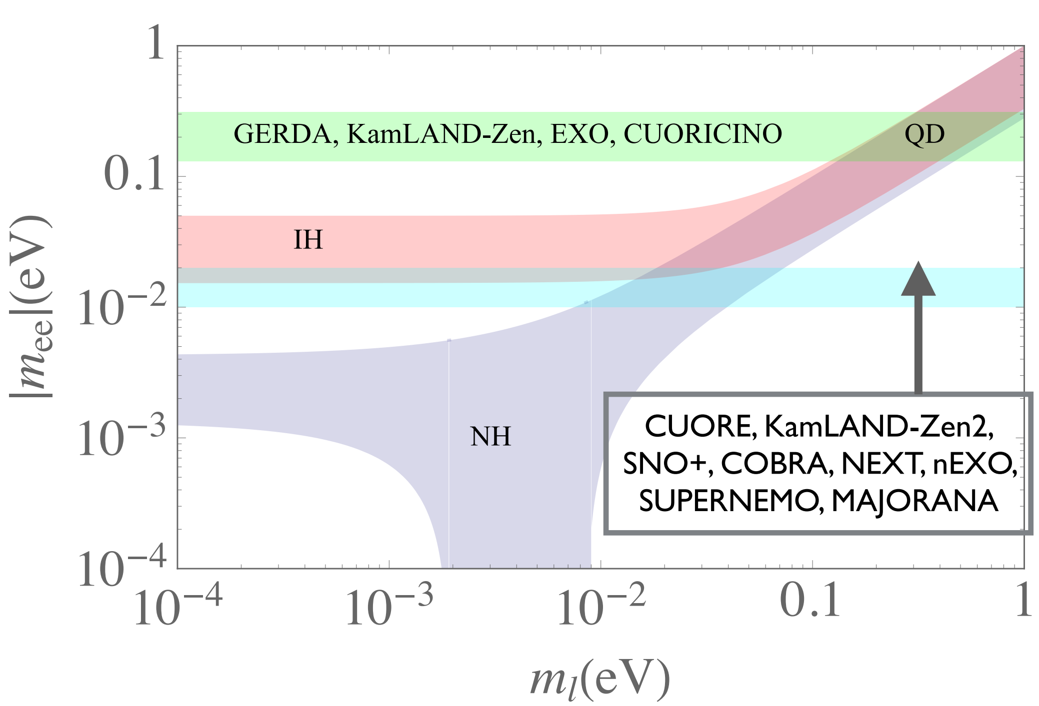
Observation of neutrinoless double beta decay would indicate the nature of the neutrino.



$$(A, Z) \longrightarrow (A, Z+2) + 2e^{-} (\Delta L = 2)$$

The half life of rare decay is proportional to effective Majorana mass:

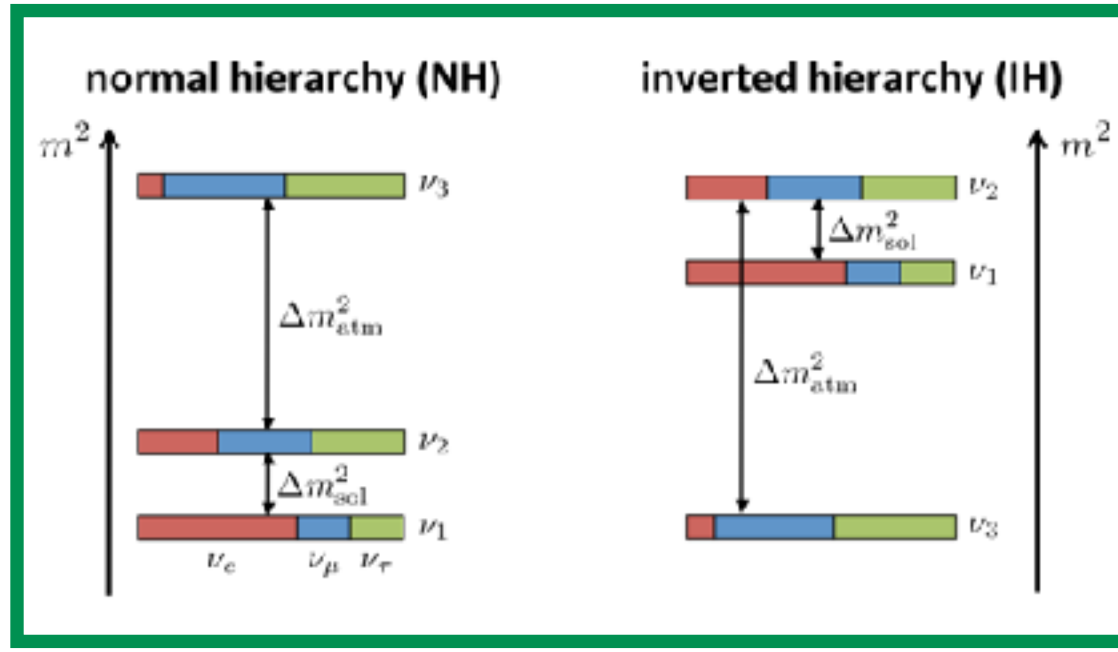
$$|m_{ee}| = |m_1 \cos^2\theta_{12} \cos^2\theta_{13} + m_2 \sin^2\theta_{12} \cos^2\theta_{13} e^{i\alpha_{21}} + m_3 \sin^2\theta_{13} e^{i(\alpha_{31}-2\delta)}|$$



Wide range of future experiments:
positive signal would mean LNV

NDBD can probe neutrino mass ordering, absolute mass scale and CPV phases (in principle).

- **How are the masses ordered?**



- **What are the precise values of the mixing angles?**
- **Is leptonic CP maximally violated?**
- **Are neutrinos Dirac or Majorana fermions?**
- **What is the mass of the lightest neutrino?**

$$\sum m_\nu \lesssim 0.2\,\mathrm{eV}$$ **cosmology, many groups see** PDG

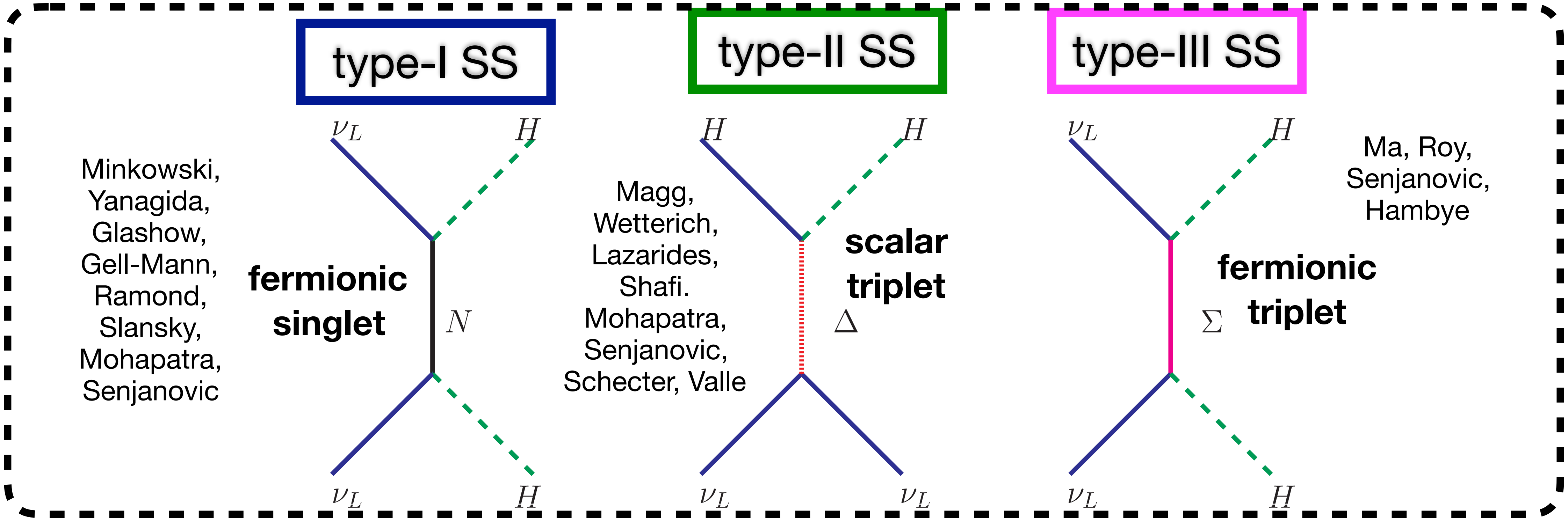
$$\sum m_\nu \leq 1.1\,\mathrm{eV}$$ **recent measurement by KATRIN** 1909.06048

We do know neutrinos are significantly lighter than other SM fermions

Neutrino Masses

- Write a Dirac mass term analogous to other SM fermions

$$-\mathcal{L}_{d=5} = \lambda \frac{L.H L.H}{M}$$



$$\mathcal{L} = -Y_\nu \overline{N} L H - \frac{1}{2} \overline{N^C} M_N N$$

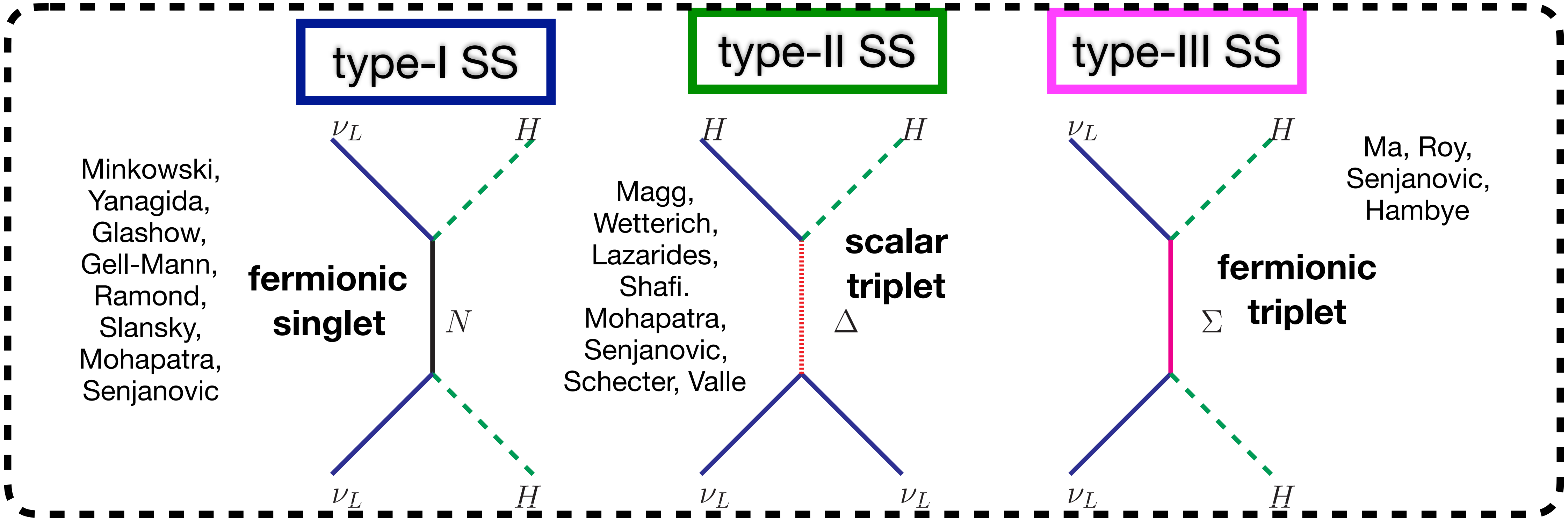
$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}$$

$$m_\nu = \frac{Y_\nu^2 v^2}{M_N} \sim 0.1\,\text{eV}$$

Neutrino Masses

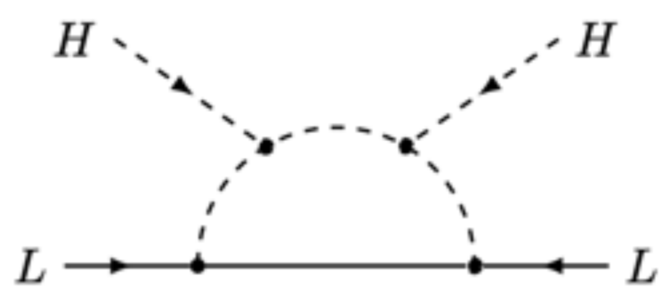
- Write a Dirac mass term analogous to other SM fermions

$$-\mathcal{L}_{d=5} = \lambda \frac{L.H\,L.H}{M}$$



- radiative models

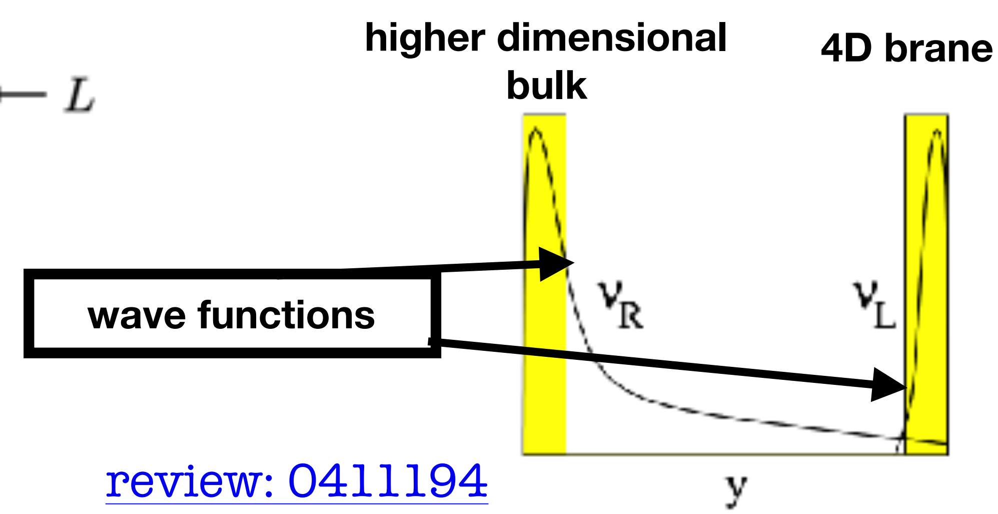
Babu, Leung, Bhattacharya, Wudka, Gouvea, Jenkins, Kobach, Ma



review: 1706.08524

- Extra dimensions

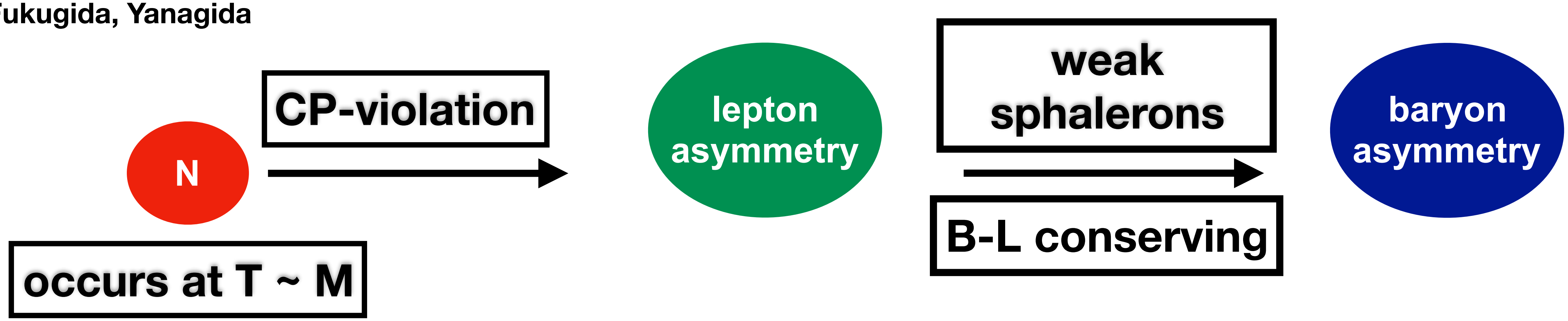
Arkani-Hamed, Dimopoulos, Dvali, March-Russell



review: 0411194

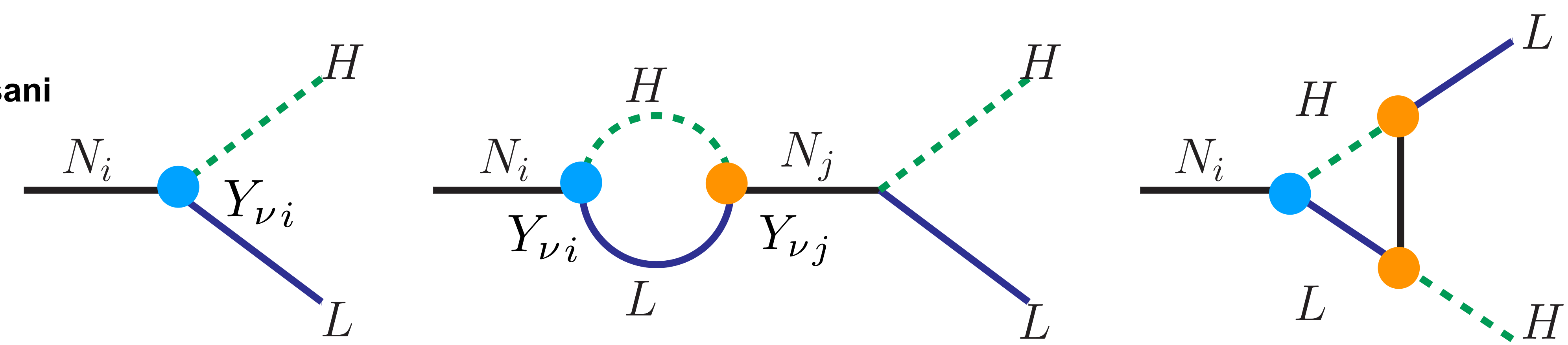
Leptogenesis via Decays

Fukugida, Yanagida



Decay asymmetry from interference between tree and loop level diagrams

Covi, Roulet, Vissani



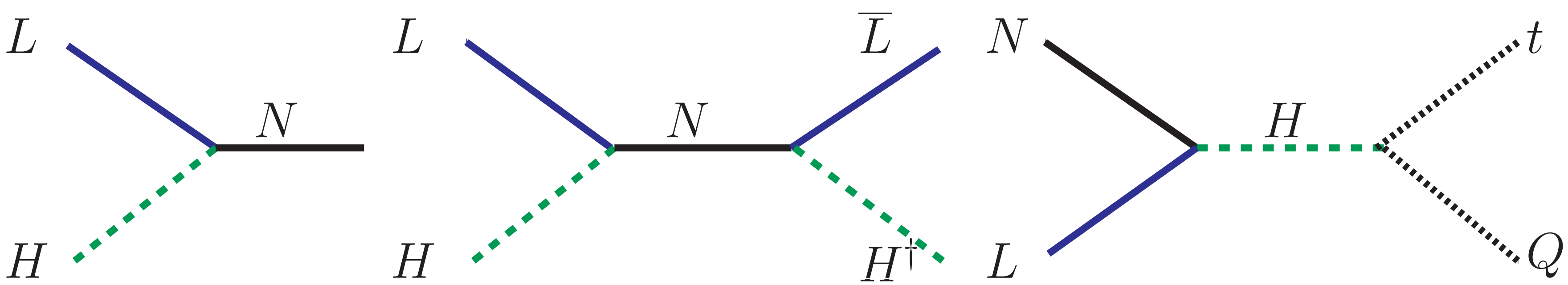
Decay Asymmetry

$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma}_i}{\Gamma_i + \overline{\Gamma}_i}$$

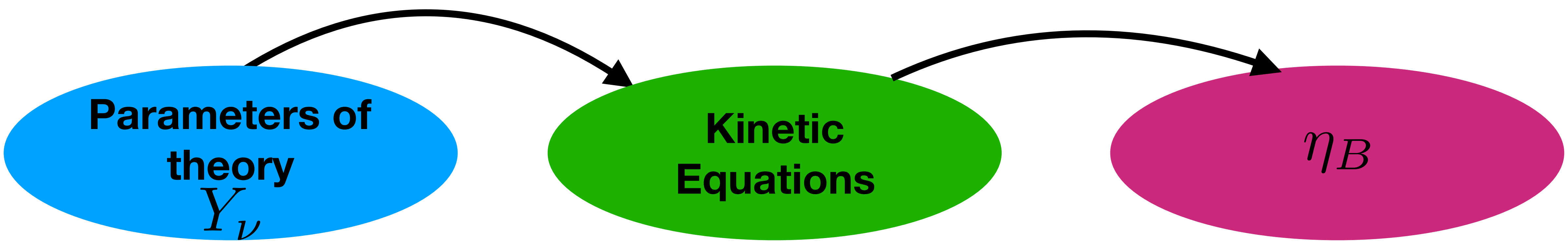
Calculation does not usually include finite density temperature effects

Basic Mechanism

Washout and Scattering processes



inverse Decays **ΔL=2** **ΔL=1**



$$\frac{dn_{N_i}}{dz} = -\,D_i(n_{N_i} - n_{N_i}^{\rm eq}),$$

source **sink**

$$\frac{dn_{B-L}}{dz} = \sum_{i=1}^{3}\left(\epsilon^{(i)} D_i(n_{N_i} - n_{N_i}^{\rm eq}) - W_i n_{B-L}\right).$$

Akmedov, Rubakov, Smirnov, Hernandez, Kekic, Lopez-Pavon, Racker, Salvado, Drewes, Garbcreht, Klaric, Gueter

Leptogenesis via oscillations

Flavour effects can lower the scale

Minimal Leptogenesis

~ eV ~ 0.1 GeV ~50 GeV ~10^5 GeV ~10^7 GeV ~10^{14} GeV

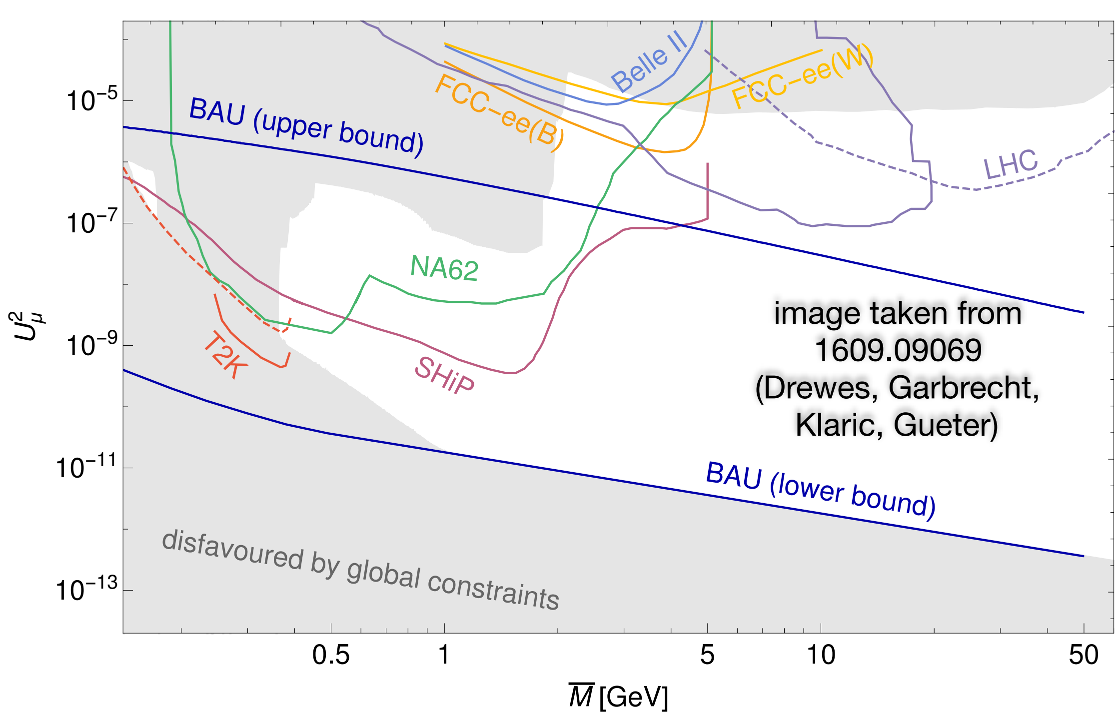
Pilaftsis, Underwood, Millington, Teresi

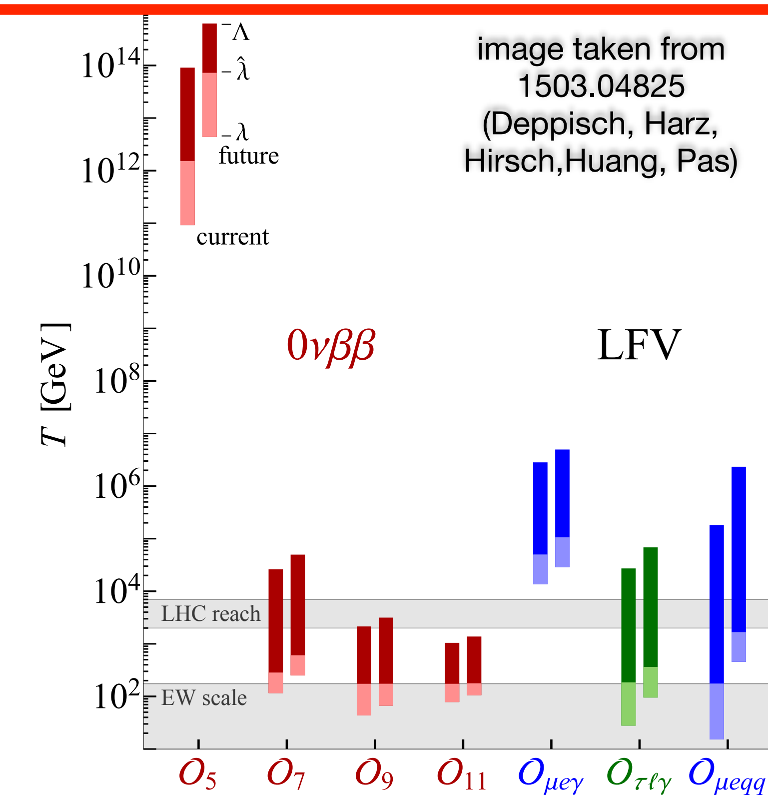
Resonant Leptogenesis

- **small neutrino masses ⟺ BAU**
- **minimal: 2RHN**
- **neutrino data, NDBD, LFV and LNV, cosmology in meson decays, collider searches**

- **small neutrino masses ⟺ BAU**
- **minimal: 2RHN**
- **Easily embedded in GUT models**
- **falsifiable**
- **Can induce the EW scale**

- **Scale too high can exacerbate**
- **Higgs fine tuning**
- **RHNs too heavy to produce**





Half time summary

- Neutrinos could be their own anti-particles

- This possibility opens up the possibility neutrino masses may be connected to the BAU.

- Neutrinos masses are much smaller than other SM fermions

- This seems to hint they may acquire their mass in a completely different way from other fermions.

- We will consider the plausibility of a rather exotic neutrino mass model….

Neutrino Masses from gravity

Dvali & Funcke (1602.03191)

Logic: make an analogue of gravity with QCD

u, d, s are light relative to c, b and t so **approximate** flavour symmetry

$$U(3)_A \times U(3)_V$$

At energies below $\Lambda_{\mathrm{QCD}} \sim 300\,\mathrm{MeV}$ quarks confine into hadrons

Once confinement occurs, relevant d.o.f baryons and mesons $\langle \overline{q} q \rangle$

Ground state break symmetry

$$U(3)_A \times U(3)_V \to U(3)_V$$

quark confinement spontaneously breaks symmetry

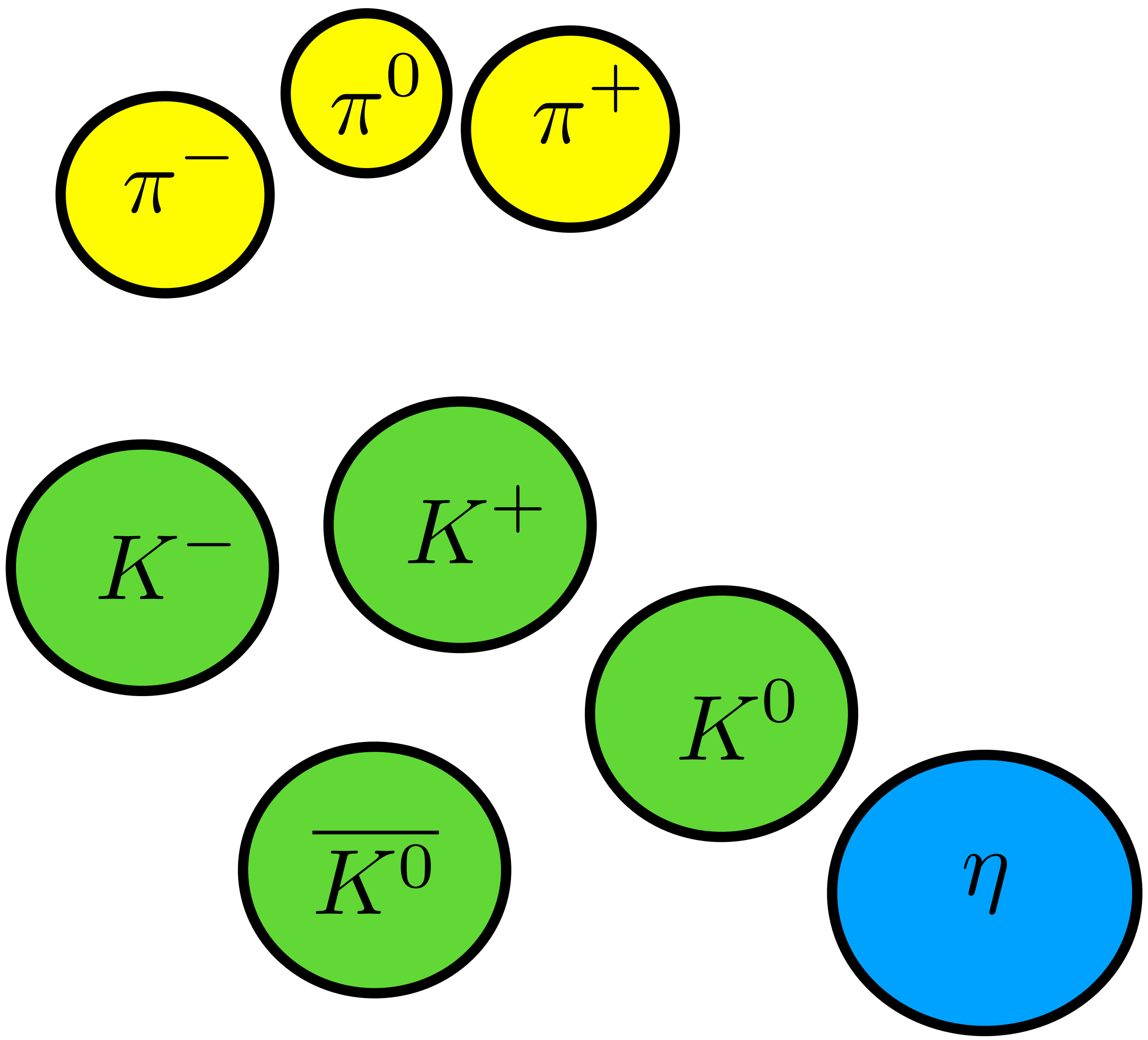
Broken symmetry contains $U(1)_A$ and an SU(3) part which are broken and via Goldstone's theorem give

$$1(\eta') + 8(\pi, \eta, K)$$

Neutrino Masses from gravity (1602.03191)

η' is heavy relative to the other mesons as its mass gets raised due to non-perturbative QCD effects.

not to scale ;)



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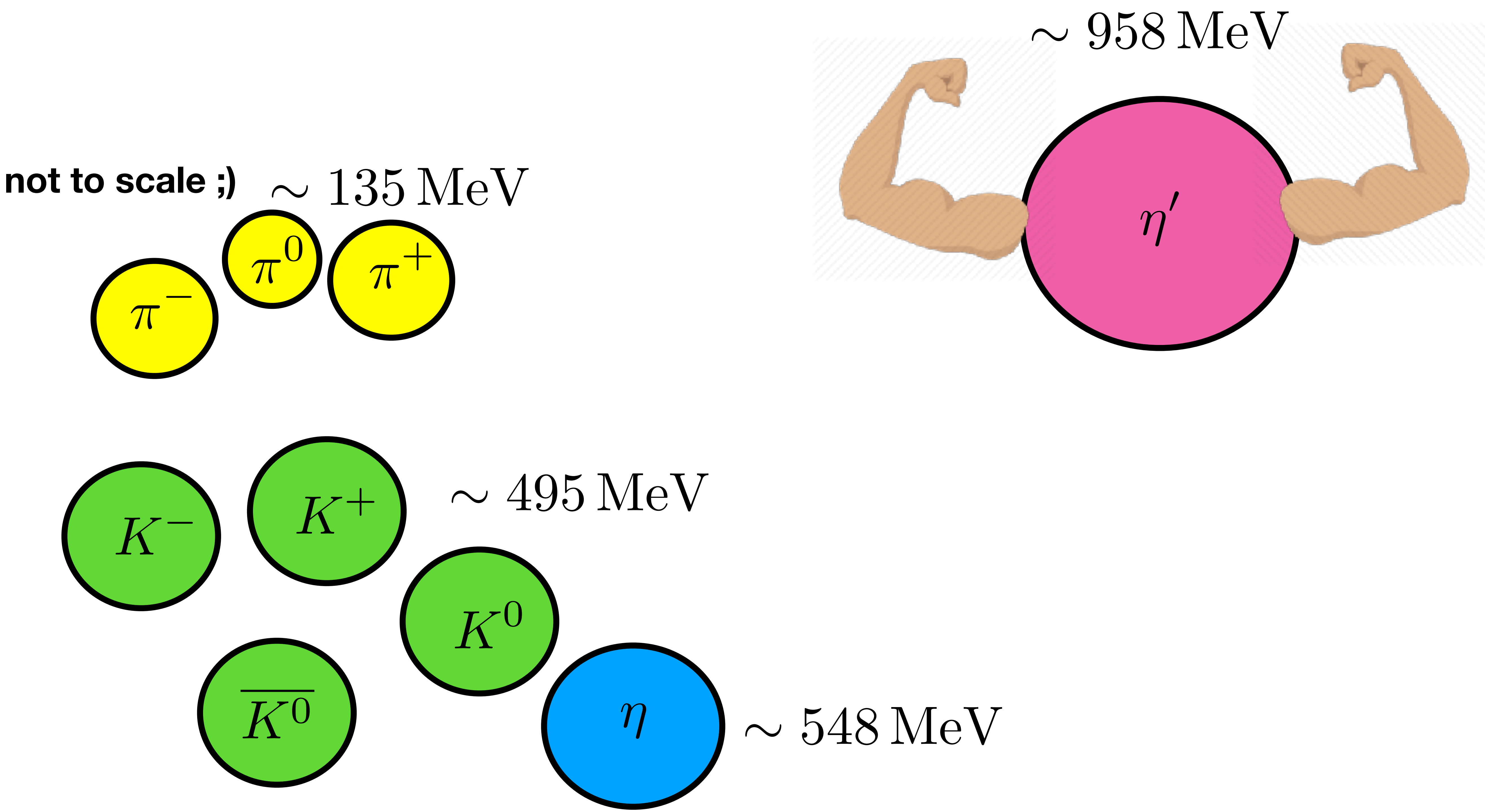

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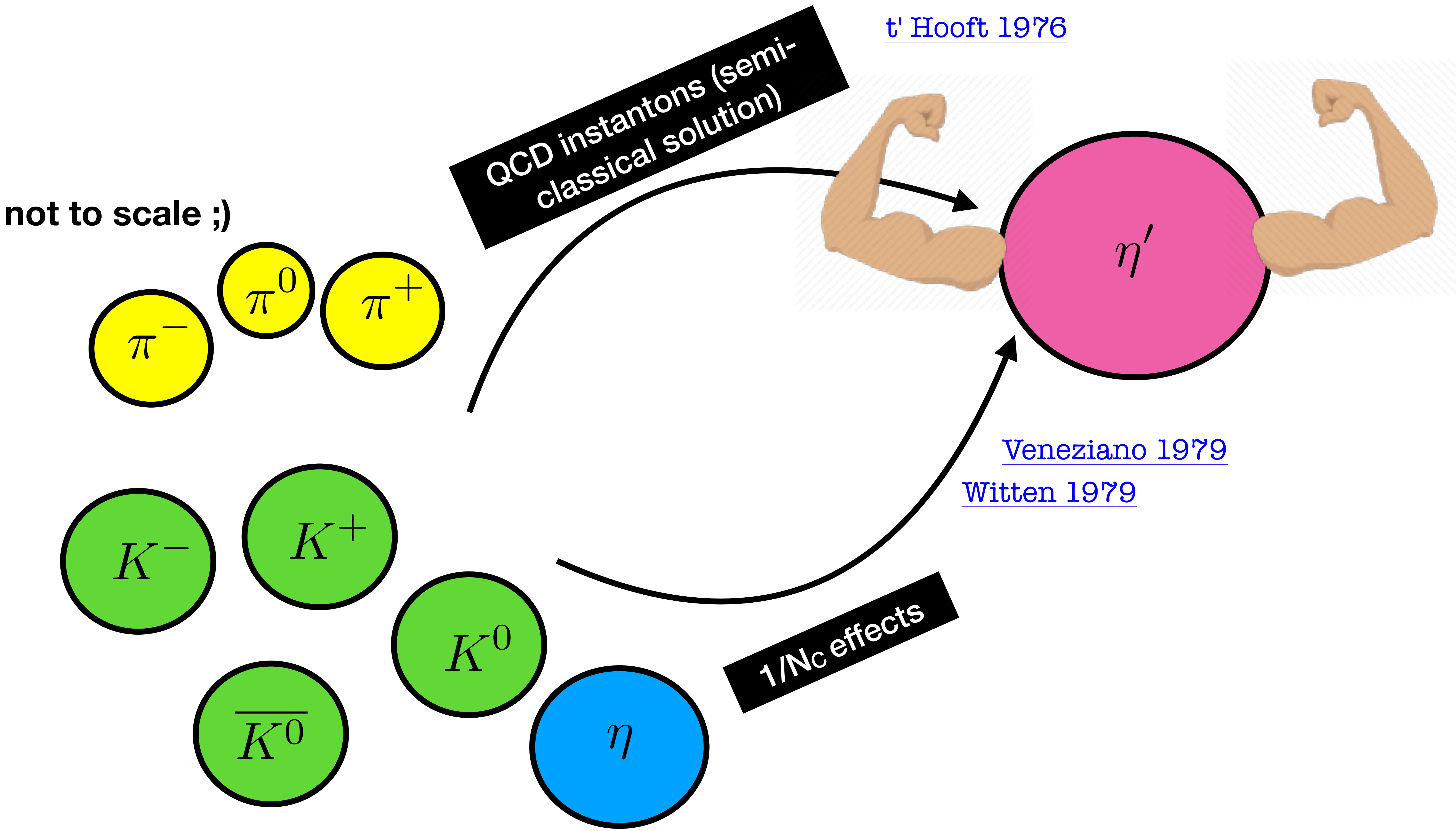

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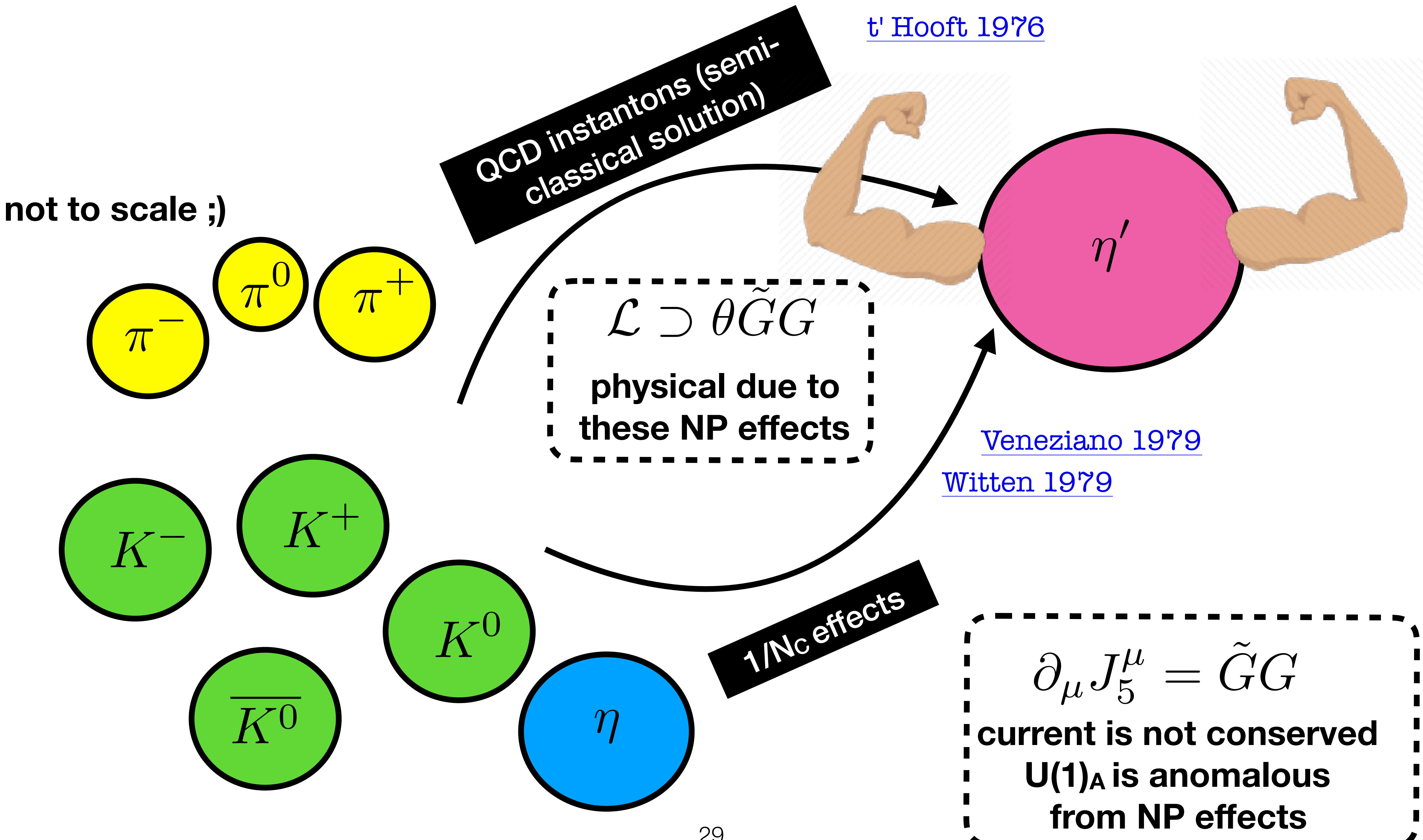


Neutrino Masses from Gravity (1602.03191)

Assume gravity has a theta term: $\mathcal{L}_G \supset \theta_G \tilde{R}R$ ← **Riemann tensor**

They postulate neutrinos have zero bare mass. They can condense via NP gravitational effects and use SDA in analogue with QCD:

$$\nu\overline{\nu} = \langle \nu\overline{\nu} \rangle = ve^{i\phi} \implies \Lambda_G \sim v \sim m_\nu \sim \nu\overline{\nu}$$

analogous to Λ_{QCD}
free parameter

$$U(3)_V \times U(3)_A \to U(1)^3$$

$$1(\eta_\nu) + 14(\phi)$$ **pseudo Goldstone bosons**

small neutrino masses from this gravitational θ term triggers neutrino condensate and introduces an **infrared gravitational scale**.

One massive GB analogous to eta prime and remainder massless

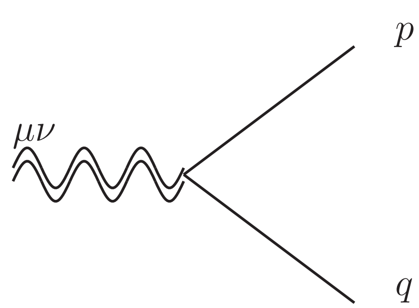
Neutrinos acquire mass from their NP coupling to neutrino condensate.

Neutrino Masses from Gravity 1909.04675

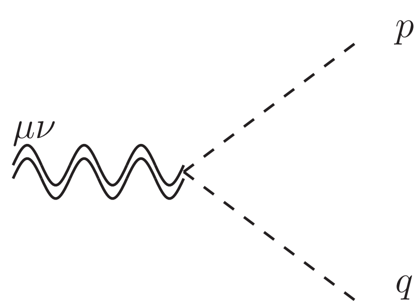
With Gabriela Barenboim (Valencia) and Ye-Ling Zhou (Southampton)

- We treat gravity as an EFT similar to Donoghue see 9405057v1 for a review. Start with flat metric and perturb around it, gravity non-Abelian gauge theory with spin-2 gauge boson.

$$g_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu}$$



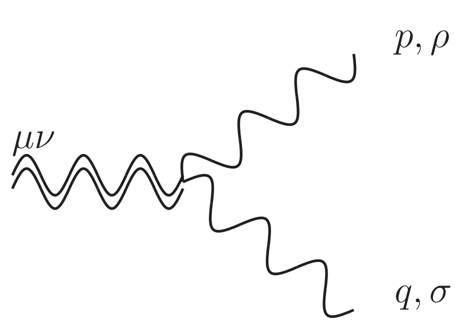
$$\tau_1^{\mu\nu}(p,q) = \frac{i\kappa}{8}\left[(q-p)^\mu \gamma^\nu + (q-p)^\nu \gamma^\mu - 2\eta^{\mu\nu}(\not{q} - \not{p})\right]$$



$$\tau_2^{\mu\nu}(p,q) = \frac{i\kappa}{2}\left[p^\mu q^\nu + p^\nu q^\mu - \eta^{\mu\nu} p \cdot q\right]$$

$$\kappa = \sqrt{32\pi G}$$

$$G = \frac{1}{M_{Pl}^2}$$



$$\tau_3^{\mu\nu\rho\sigma}(p,q) = i\kappa[-\mathcal{P}^{\mu\nu\rho\sigma} - \frac{1}{2}\eta^{\mu\nu} p^\sigma q^\rho + \eta^{\sigma\rho}(p^\mu q^\nu + p^\nu q^\mu)$$

$$+ \frac{1}{2}(\eta^{\mu\sigma} p^\nu q^\rho + \eta^{\nu\sigma} p^\rho q^\mu + \eta^{\nu\rho} p^\sigma q^\mu + \eta^{\mu\rho} p^\nu q^\sigma)]$$

$$\mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$$

Neutrino Masses from Gravity [1909.04675](1909.04675)

With Gabriela Barenboim (Valencia) and Ye-Ling Zhou (Southampton)

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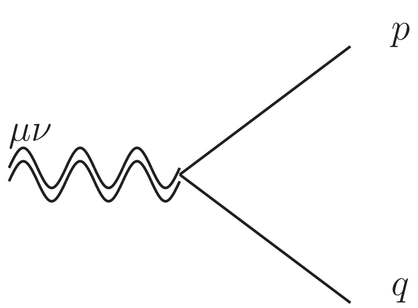
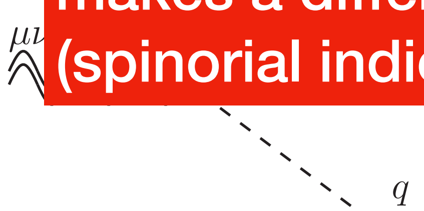
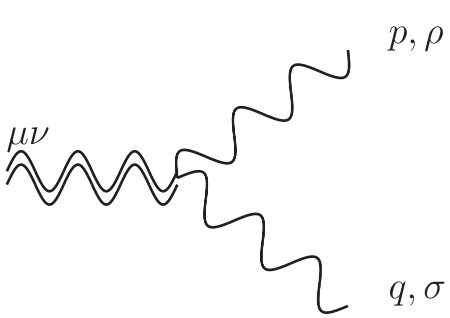


$$\tau_1^{\mu\nu}(p,q) = \frac{i\kappa}{8}\left[(q-p)^\mu\gamma^\nu + (q-p)^\nu\gamma^\mu - 2\eta^{\mu\nu}(\not{q} - \not{p})\right]$$

This 2 is often missing in the literature, it should be there! It makes a difference! Problem comes from embedding fermions (spinorial indices) into GR.

$$\kappa = \sqrt{32\pi G}$$
$$G = \frac{1}{M_{Pl}^2}$$





$$\tau_3^{\mu\nu\rho\sigma}(p,q) = i\kappa\Big[-\mathcal{P}^{\mu\nu\rho\sigma} - \frac{1}{2}\eta^{\mu\nu}p^\sigma q^\rho + \eta^{\sigma\rho}(p^\mu q^\nu + p^\nu q^\mu) + \frac{1}{2}(\eta^{\mu\sigma}p^\nu q^\rho + \eta^{\nu\sigma}p^\rho q^\mu + \eta^{\nu\rho}p^\sigma q^\mu + \eta^{\mu\rho}p^\nu q^\sigma)\Big]$$

$$\mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$$

- “Gravity triggered neutrino condensates” (1009.2504) used SDEs as a means of studying RH neutrino condensation. We revisit this paper and calculation techniques for light neutrinos.

- Here RHN are heavy and light neutrinos get mass from type-I ss. RHN condensate can drive inflation (0811.2998)

- SDE are an infinite tower of integral coupled equations which relate the Green’s functions of a theory to each other.

- We have to choose some truncation skim. For us this is one loop improved.

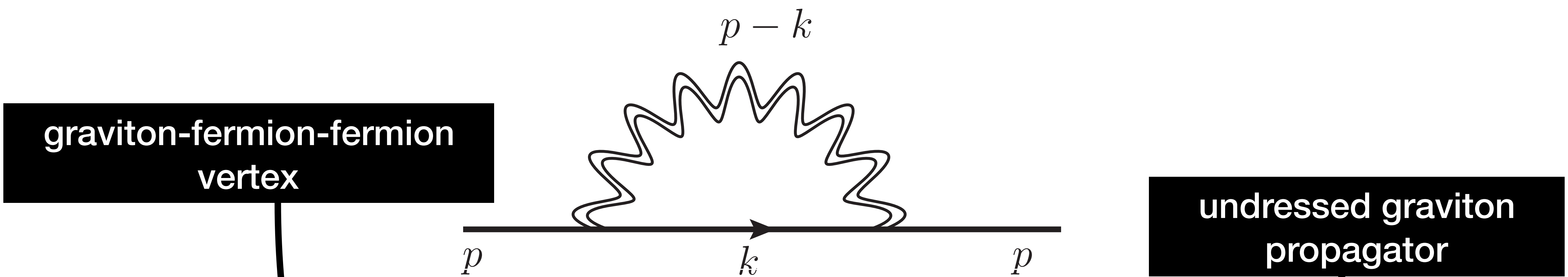
- This allows us to derive a neutrino gap equation.

Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

$$S'_F(p) = \frac{i}{\not{p} - \Sigma(p)} = \frac{i}{\alpha(p^2)\not{p} - \beta(p^2)} \qquad m_F = \beta(p^2)/\alpha(p^2)$$

Assume neutrino has zero valued bare mass and Dirac fermion.



Leading self-energy correction

$$-i\Sigma(p) = \int \frac{d^4k}{(2\pi)^4} \tau_1^{\mu\nu}(p,-k) S'_F(k) G_{\mu\nu\rho\sigma}(p-k) \tau_1^{\rho\sigma}(k,-p)$$

Neutrino Masses from Gravity

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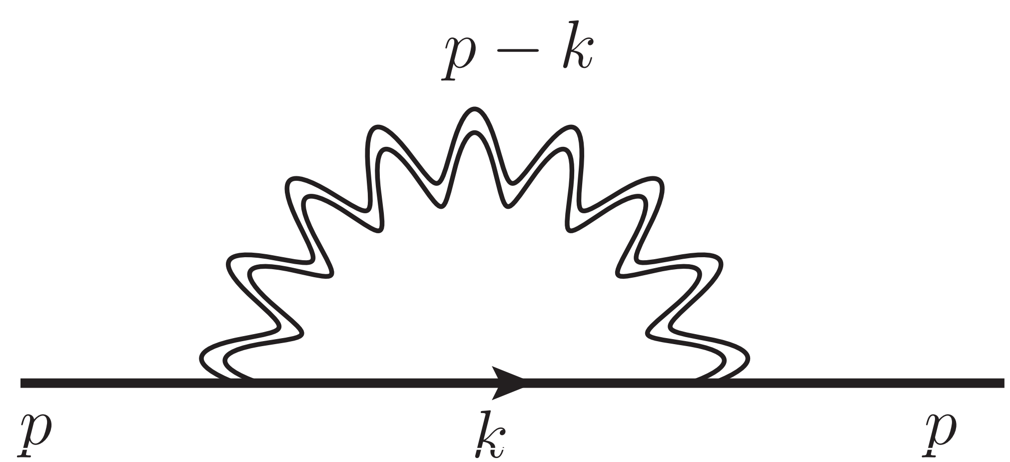

Taking the appropriate Dirac trace

$$\alpha(p^2) = 1 - \frac{1}{4p^2}\mathrm{tr}(\not{p}\Sigma(p)) \quad \beta(p^2) = \frac{1}{4}\mathrm{tr}(\Sigma(p))\,.$$

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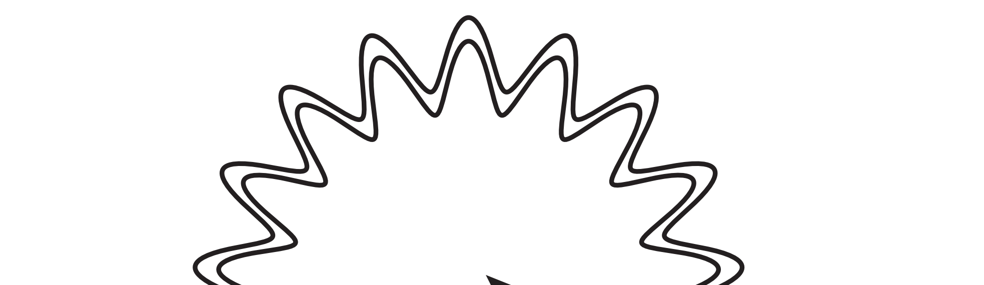

$$\beta(p^2) = 0 \ \forall p$$

$$\alpha(p^2) = 1 - i2\pi G \int \frac{d^4k}{(2\pi)^4} \frac{\alpha(k^2)}{\alpha^2(k^2)k^2 - \beta^2(k^2)} \frac{\left[2(k\cdot p)^2 + 4k^2p^2 + 3k\cdot p(k^2+p^2)\right]}{p^2(p-k)^2}$$

Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

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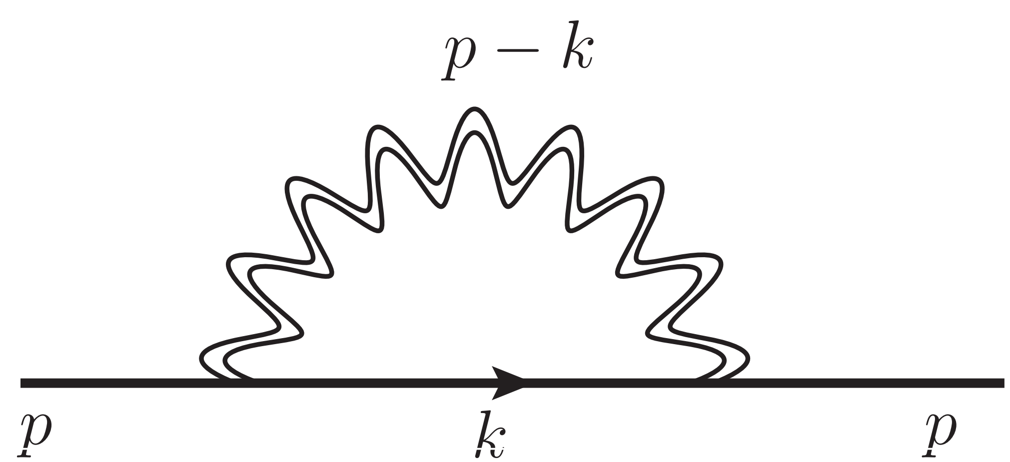
Assume neutrino has zero valued bare mass. Leading contribution from undressed graviton propagator is vanishing. Need to dress graviton.

no mass dynamically induced if graviton is undressed



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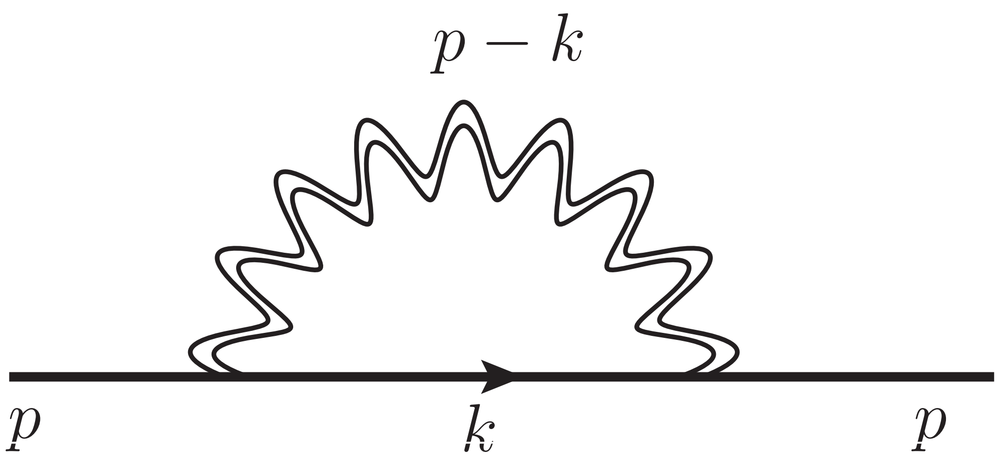

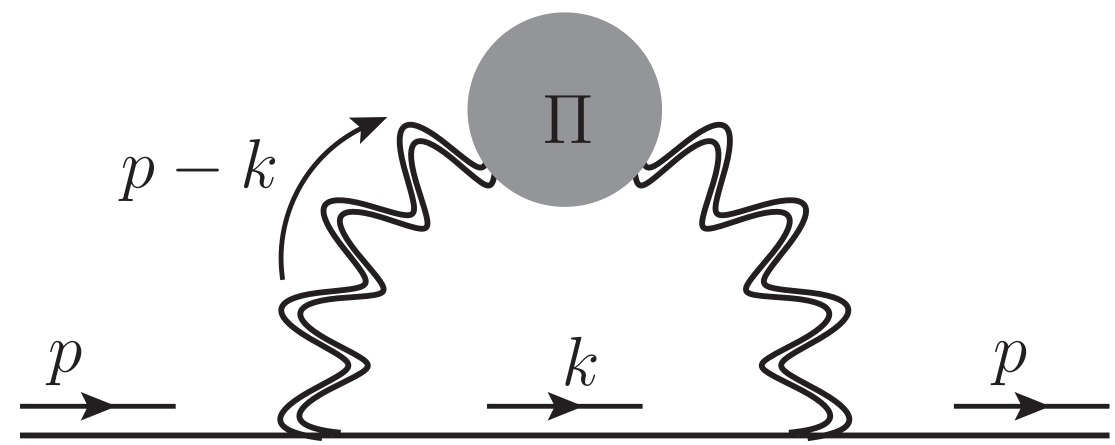

$$G'_{\mu\nu\rho\sigma}(p-k) \to G_{\mu\nu\rho\sigma}(p-k) + G_{\mu\nu\alpha\beta}(p-k)\Pi^{\alpha\beta,\gamma\delta}(p-k)G_{\rho\sigma\gamma\delta}(p-k)$$

Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

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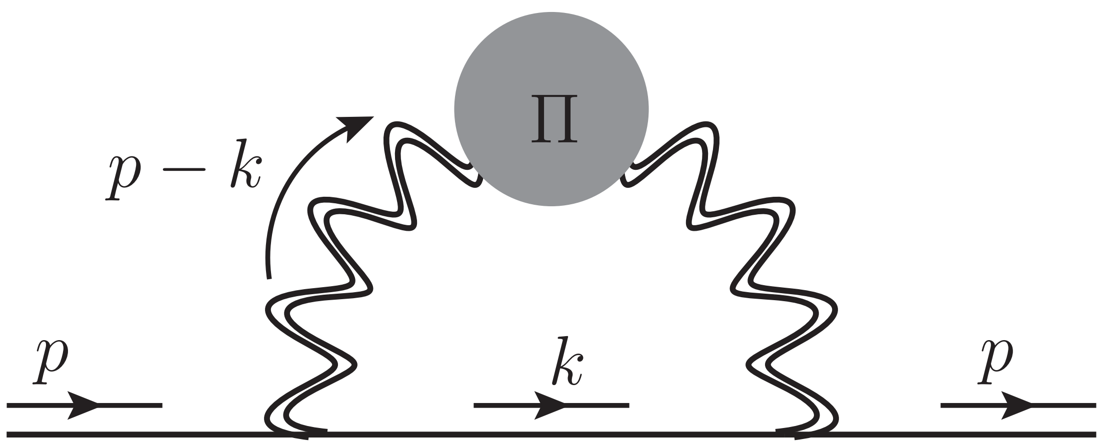
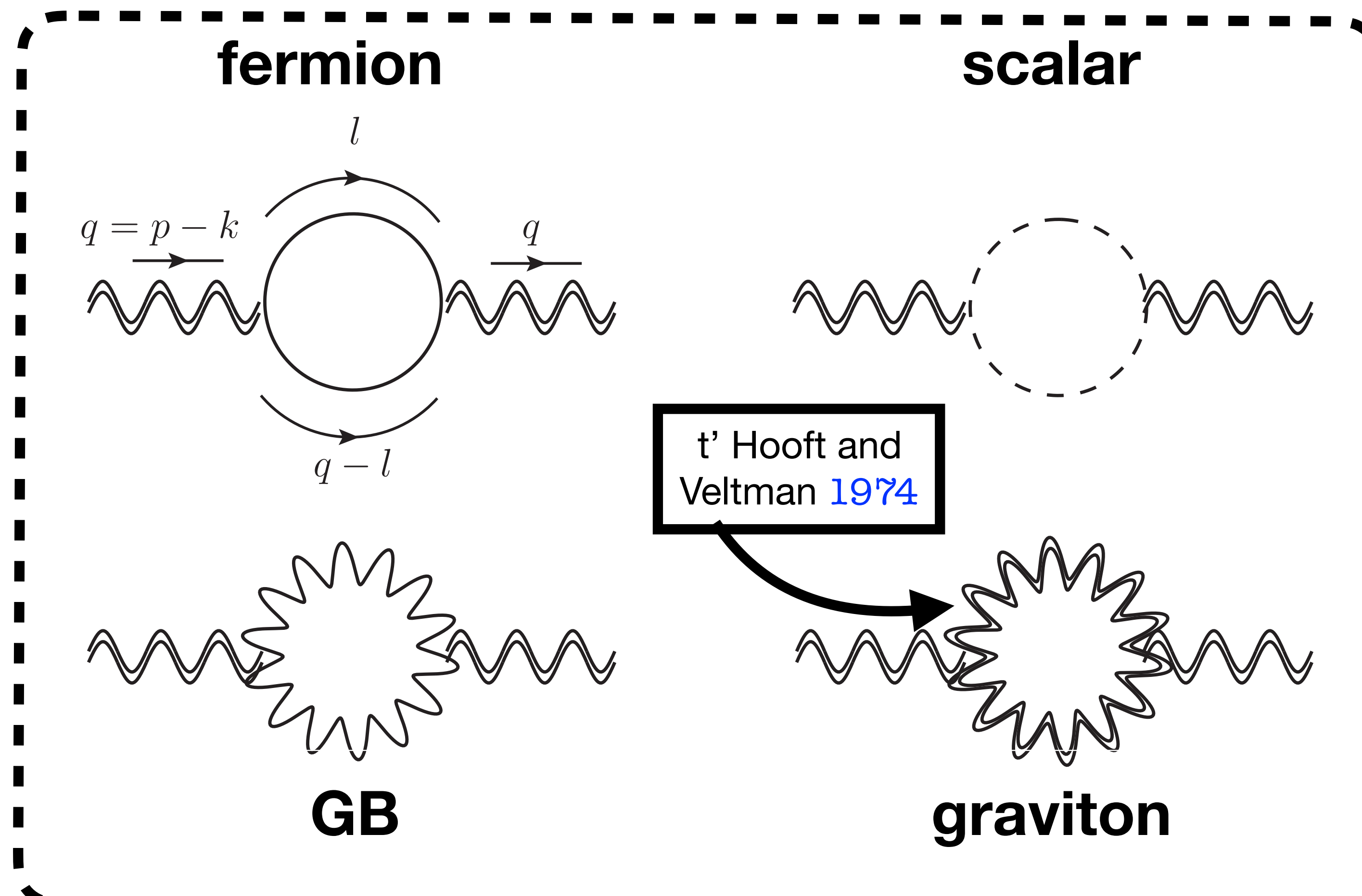


Vacuum polarisations Π



Neutrino Masses from Gravity

Rotate to Euclidean space and rescale momenta: $x = \frac{p_E^2}{\Lambda^2} \quad y = \frac{k_E^2}{\Lambda^2}$

$$\alpha(p_E^2) = \quad 1 - 2\pi G \int \frac{d^4 k_E}{(2\pi)^4} \frac{\alpha(k_E^2)}{\alpha^2(k_E^2)k_E^2 + \beta^2(k_E^2)} \frac{\left[2(k_E \cdot p_E)^2 + 4k_E^2 p_E^2 + 3k_E \cdot p_E(k_E^2 + p_E^2)\right]}{p_E^2(p_E - k_E)^2}$$

$$\beta(p_E^2) = \quad -8G^2 \int \frac{d^4 k_E}{(2\pi)^4} \frac{\beta(k_E^2)}{\alpha^2(k_E^2)k_E^2 + \beta^2(k_E^2)} \left[A(k_E + p_E)^2 - B\frac{\left(p_E^2 - k_E^2\right)^2}{8(p_E - k_E)^2}\right] \log\left[\frac{\mu^2}{(p_E - k_E)^2}\right]$$

UV cutoff

$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2} \int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x) + \beta^2(y)} K(x, y)$$

$$\beta(x) = \frac{8G^2\Lambda^4}{(2\pi)^3} \int_0^1 dy \frac{y\beta(y)}{y\alpha^2(y) + \beta^2(y)} L(x, y)$$

Kernels

Kernel structure

Rotate to Euclidean space, rescale momentum

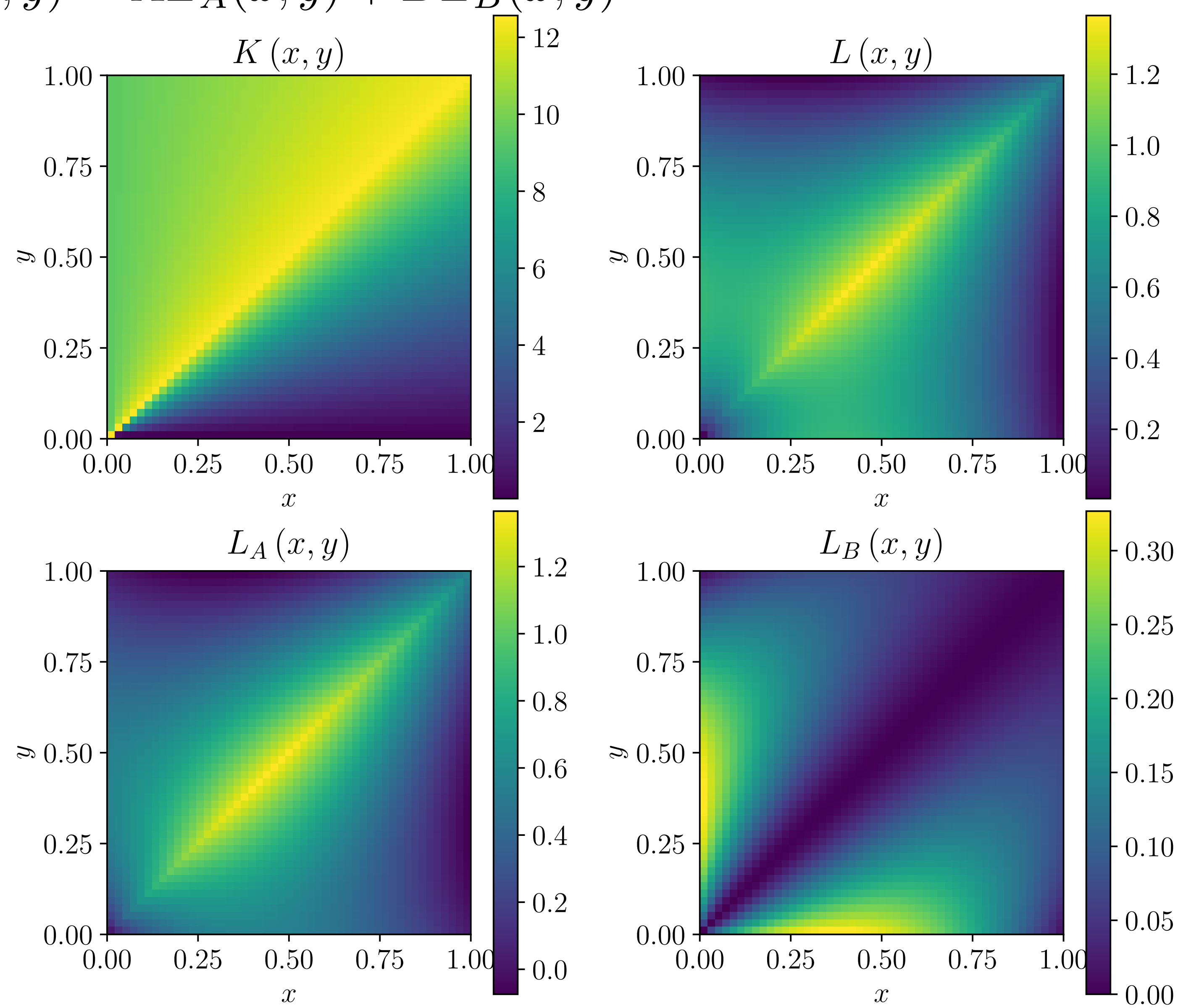
rescaled p momentum

$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2} \int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x) + \beta^2(y)} K(x, y)\,,$$

rescaled k momentum

$$\beta(x) = \frac{8G^2\Lambda^4}{(2\pi)^3} \int_0^1 dy \frac{y\beta(y)}{y\alpha^2(y) + \beta^2(y)} L(x, y)\,.$$

$L(x,y) = AL_A(x,y) + BL_B(x,y)$



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Kernel structure

Rotate to Euclidean space, rescale momentum

rescaled p momentum

$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2}\int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x)+\beta^2(y)} K(x,y)\,,$$

rescaled k momentum

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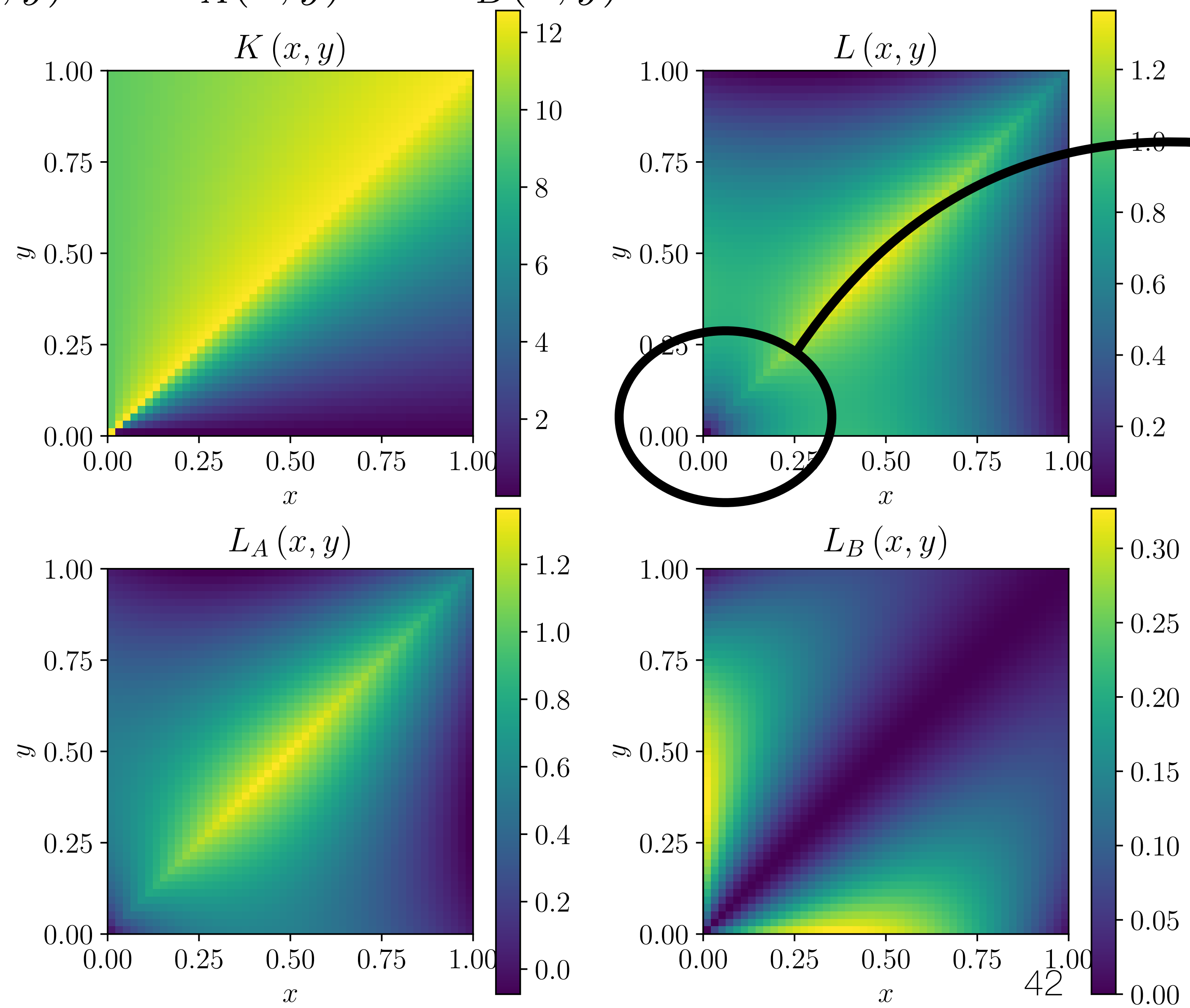


gravity $L(0,0)=0$
$x=y=0$

QCD $L(0,0)\neq 0$
$x=y=0$

d-wave interaction due to spin 2 nature of graviton

42
42

Use two methods to solve SDEs using two well known methods

1. **Solve equations iteratively and apply extrapolation**

2. **Make informed Ansatz of the kernel and check for self consistency of non-trivial vacuum.**

Solving SD Equation - Extrapolation

$$\alpha^{(i+1)}(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2}\int_0^1 dy \frac{y\alpha^{(i)}(y)}{y{\alpha^{(i)}}^2(x) + {\beta^{(i)}}^2(y)} K(x,y)$$

$$\beta^{(i+1)}(x) = \frac{8(G\Lambda^2)^2}{(2\pi)^3}\int_0^1 dy \frac{y\beta^{(i)}(y)}{y{\alpha^{(i)}}^2(y) + {\beta^{(i)}}^2(y)} L(x,y)\,.$$

Start with two trial functions

$$\alpha^{(0)}(x) = c_1\,, \quad \beta^{(0)}(x) = c_2$$

$$\text{tolerance} \equiv \frac{\beta^{(i+1)}(x)}{\beta^{(i)}(x)} - 1$$

This method allows us to find the NP non-trivial vacuum.

Solution (true non-trivial vacuum) is not sensitive to trial function value or tolerance value.

Solving SD Equation - Extrapolation

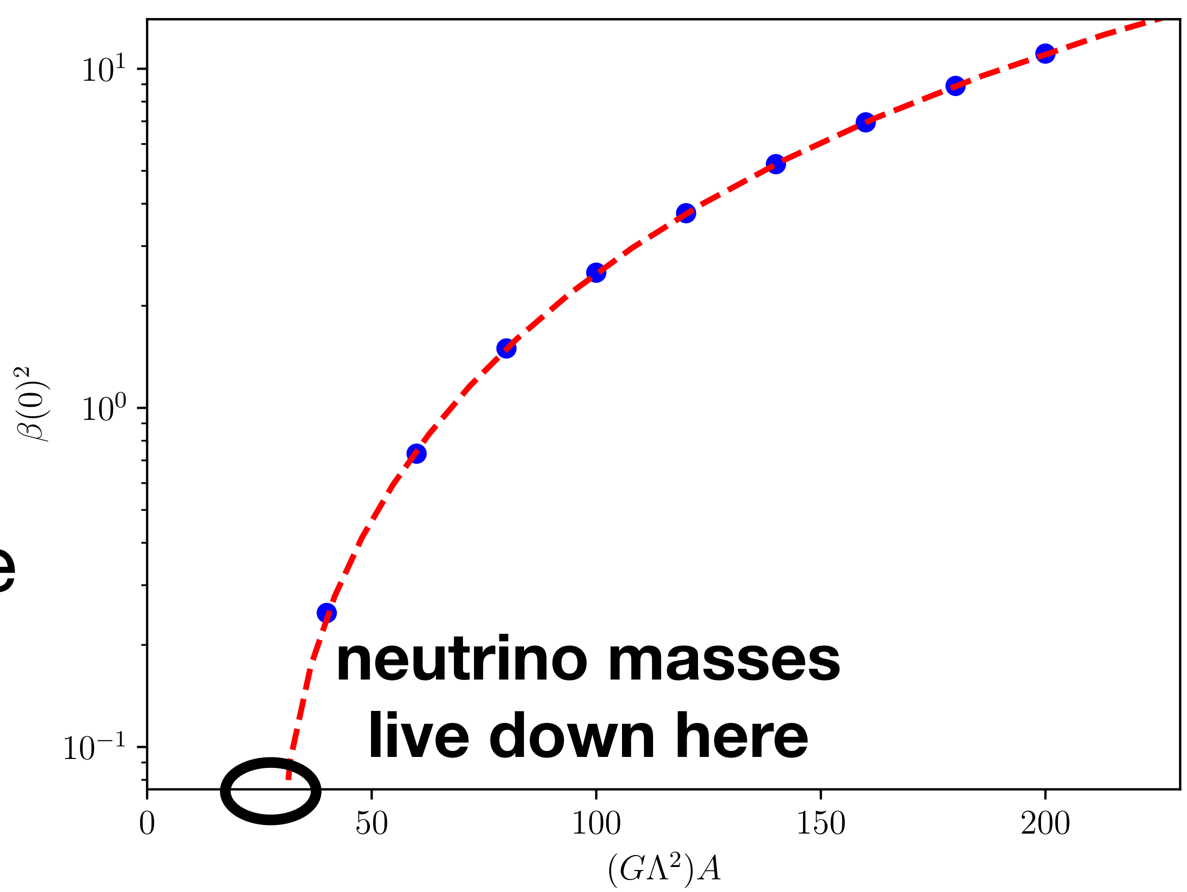
1. Choose a value of $G\Lambda^2$, A, B, tolerance and trial function values.
2. Subdivide the x-interval $[x_{\mathrm{IR}}, 1]$ into n bins, where x_{IR} is infrared boundary of the theory.
3. Iteratively solve SDE for each bin.
4. For each bin calculate the tolerance and summate this measure over all bins.
5. Require the tolerance to be close to 0.0. For example, for $G\Lambda^2 = 1.0$, $A = 50.0$ and $B = 43.5$ we choose a tolerance of 10^{-6}.

$$m_\nu = \frac{\beta(0)}{\alpha(0)}\Lambda$$

$$\beta(0) \approx 10^{-29} \text{ for } \Lambda \approx M_{\mathrm{pl}}$$

$$G\Lambda^2 \implies A \gtrsim 23$$

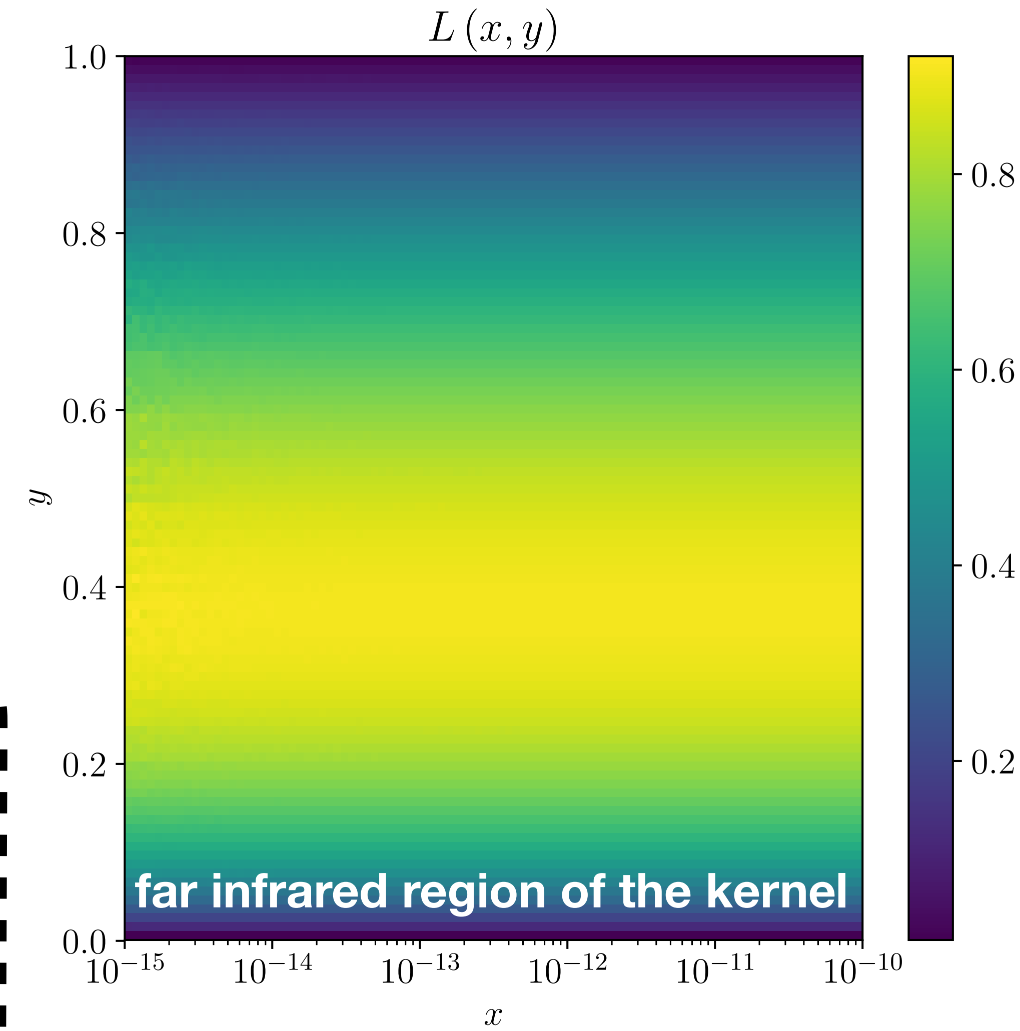
Requires beyond SM particle content to support the condensate even if scale is high, also we are tuning around chiral preserving point.



Solving SD Equation - Ansatz

Take quenched limit for simplicity i.e $\alpha \approx 1$

β depends on L(x,y). This kernel is flat in the x-direction even for tiny momentum. Make Ansatz that β is a step function of magnitude ‘a’.

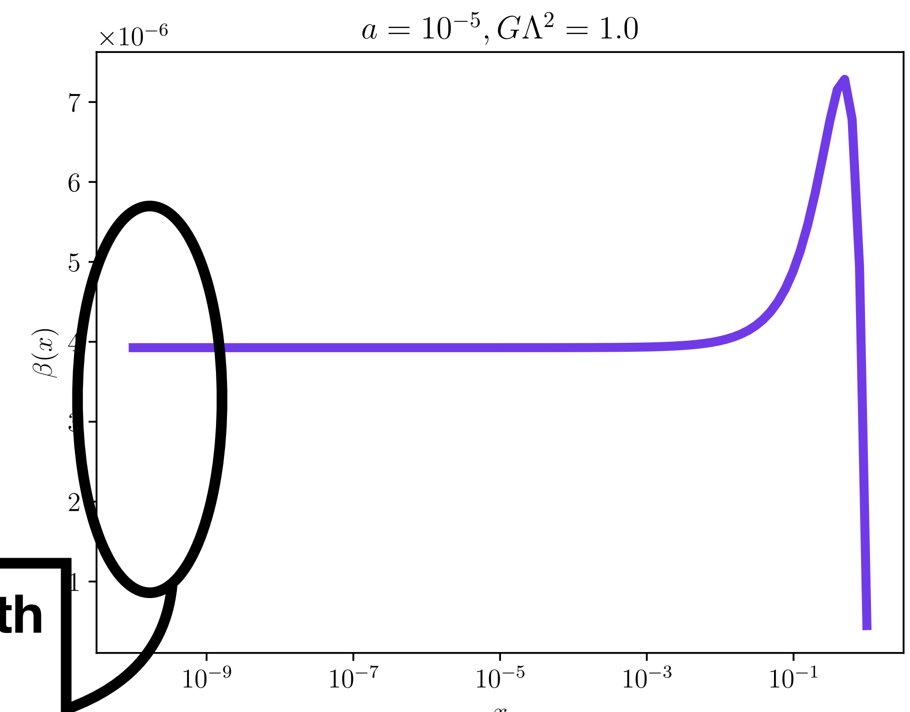


$$\beta(x) = \frac{8(G\Lambda^2)^2}{(2\pi)^3} \int_0^1 \frac{a y dy}{a^2 + y} L_A(x, y)$$

$$A L_A(x, y) \gg B L_B(x, y)$$

Solve SDE for this Ansatz

Checks for self-consistency of postulated form of β with kernel structure.
As before the mass can be tuned but this this approach it is tuned via ‘a’.



concerned with the deep IR neutrino mass

Summary

- Neutrinos are unique amongst the Standard Model (SM) fermions in the tininess of their mass, the weakness of their interactions and their capacity to be their own anti-particles. Such features suggest neutrinos acquire their mass in a different way from the quarks and charged leptons.

- Neutrino masses from gravity is an intriguing idea and we have made a first calculational attempt at exploring this possibility.

- An interesting feature is new d.o.fs are necessary to provide finite support to the condensate even if it occurs at a very high scale. SM + gravity is not sufficient unless there are large ED which lowers Planck scale.

- As gravity does not discriminate between the neutrinos, they are mass degenerate, one needs some additional mechanism to induce a mass splitting.

- However a high level of fine-tuning is required if the Planck scale is at $\sim 10^{19}$ GeV.

Thank you for your attention

Back up slides - the action

$$S_g = \int d^4x\sqrt{-g}(\frac{1}{4\pi G}R + \mathcal{L}_m)$$

$$\mathcal{L}_m = D_\mu\phi^* g^{\mu\nu} D_\nu\phi + \frac{i}{2}\left[\bar{\psi}\gamma^a e^\mu_a D_\mu\psi + (D_\mu\bar{\psi})\gamma^a e^\mu_a\psi\right] - \frac{1}{4}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}$$

where $F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu$ and D_μ denotes the covariant derivative with respect to the gravitational field and gauge fields, and e^μ_a is the vierbein to shift frame to the local Minkowski flat frame.

$$g_{\mu\nu} \rightarrow \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

Perturb the metric, the classical gravitational field is fixed at zero.

$$\text{Graviton propagator} : G_{\mu\nu\rho\sigma}(p) = \frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{p^2}$$

$$\mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$$

Back up slides - kernel structure

$$K(x,y) = \frac{1}{x}\int_0^{\pi} \sin^2\theta d\theta \frac{2xy\cos^2\theta + 4xy + 3\sqrt{xy}(x+y)\cos\theta}{x+y-2\sqrt{xy}\cos\theta}$$

$$L(x,y) = \int_0^{\pi} \sin^2\theta d\theta \left[A\left(x+y+2\sqrt{xy}\cos\theta\right) - B\,\frac{(x-y)^2}{8(x+y-2\sqrt{xy}\cos\theta)}\right]\times\log\left[x+y-2\sqrt{xy}\cos\theta\right]$$

$$K(x,y) = \frac{\pi}{x}\frac{(x+y)^3 - [(x+y)^2 + 2xy]|x-y|}{2xy}\,,$$

$$L_A(x,y) = \frac{\pi}{12}\left\{\frac{5(x^2+y^2) - 5(x+y)|x-y| - 6xy}{(x+y)+|x-y|} - 6(x+y)\log\left[\frac{(x+y)+|x-y|}{2}\right]\right\}\,,$$

$$L_B(x,y) = \frac{\pi}{8}\frac{(x-y)^2}{xy}\left\{\frac{(x+y)-|x-y|}{2} - \frac{(x+y)+|x-y|}{2}\log\left[\frac{(x+y)+|x-y|}{2}\right]\right.$$

$$\left. + |x-y|\log(|x-y|)\right\}\,,$$

Back up slides -regularisarion procedure

- Simplest procedure is cutoff regularisation. At energies above the cutoff the neutrino is massless and condensate dissolves.
- It has it’s ugly features, we agree: lack of Lorentz covariance, but is numerically convenient and used frequently in QED and QCD SDE.
- For QED it has been demonstrated the qualitative difference is not significant so we proceed, see e.g. 9410286 or 9604402

massless fermions and symmetries

$$\mathcal{L} = i\overline{\Psi_j}\gamma^\mu\partial_\mu\Psi_j$$

<u>**vector transformation**</u>

$$\Psi \to e^{-i\frac{\tau}{2}\phi}\Psi$$

$$\overline{\Psi} \to e^{i\frac{\tau}{2}\phi}\overline{\Psi}$$

$$i\overline{\Psi_j}\partial_\mu\Psi_j \xrightarrow[\text{vector}]{\text{transformation}} i\overline{\Psi_j}\partial_\mu\Psi_j$$

<u>**axial transformation**</u>

$$\Psi \to e^{-i\gamma_5\frac{\tau}{2}\phi}\Psi$$

$$\overline{\Psi} \to e^{i\gamma_5\frac{\tau}{2}\phi}\overline{\Psi}$$

$$i\overline{\Psi}_j\gamma^\mu\partial_\mu\Psi_j \xrightarrow[\text{axial}]{\text{transformation}} i\overline{\Psi}_j\gamma^\mu\partial_\mu\Psi_j$$

$\delta\mathcal{L} = m\overline{\Psi}\Psi$ **is not invariant under axial symmetry transformation**