Precision physic at colliders: introducing reSolve , a transverse momentum resummation tool

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1. A case for precision

The SM, the Higgs and Beyond

- We all know this story: we turned on LHC hoping to find a lot of New Physics
- But only found the Higgs (which is very interesting, but still)
- What now?

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The SM, the Higgs and Beyond

- We have several good reasons (hierarchy problem, neutrino masses, strong CP violation, Dark Matter . . .) to believe the SM has to be extended
- The leading principle (arguably) has been the hierarchy problem. But did it mislead us?
- A common (necessarily) feature of surviving BSM models is "decoupling": deviations from SM can be made small by accepting stronger amounts of fine tuning

The SM, the Higgs and Beyond

- Hierarchy, fine-tuning: they are qualitative statements – no clear point to draw a line
- Also, the SM arguably should really be seen as an EFT (a good amount of work lately about this)
- Two statements, same conclusion: deviations from the SM will appear at a certain point...
(possibly so late as to be meaningless to us, hopefully not)
- The only way to know is for experiments to measure as many observables as we can as precisely as we can. And theorists have to keep up!

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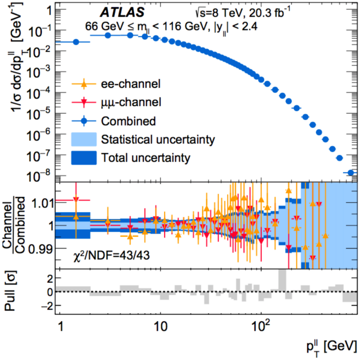
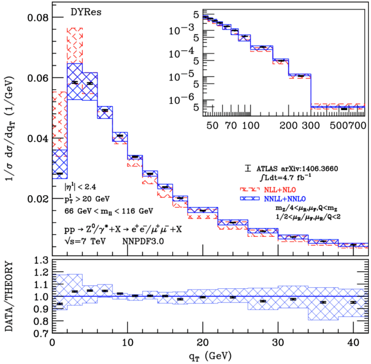
2. Transverse momentum (q_T) spectrum

q_T spectrum: general motivations

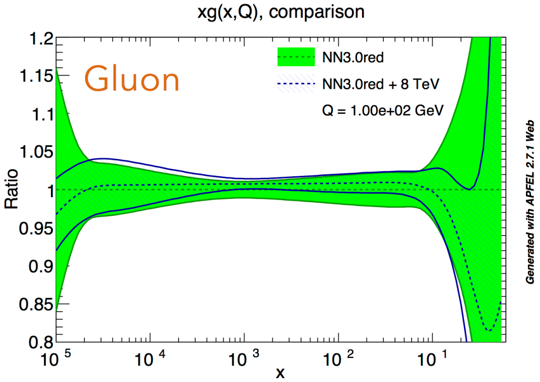
- Experimentals can measure it...
  and they are much more precise than theorists!





q_T spectrum: general motivations

- It is the most standard way of measuring m_W
- Can be used to reduce uncertainty on PDFs



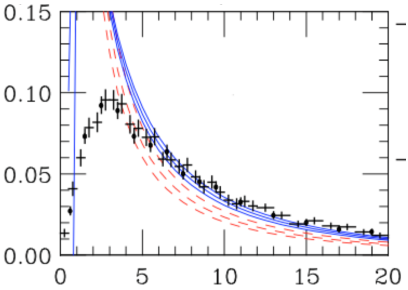
q_T spectrum: general motivations

- ...this can actually have serious consequences for phenomenology – for the Higgs for instance:

	Before ZpT data	After ZpT data
H(ggF)	48.22 ± 0.89 (1.8%)	48.61 ± 0.61 (1.3%)
H(VBF)	3.92 ± 0.06 (1.5%)	3.96 ± 0.04 (1.0%)

q_T spectrum: resummation

- Cross-section: a great deal of events are in a relatively soft q_T region



- This requires resummation – main topic of the talk

3. Theory overview for resummation

Why resummation?

- Consider a generic inclusive $h_1 h_2 \to F + X$ process @ a hadron collider

- Use standard factorization for the q_T distribution:

$$\frac{d\sigma^F}{dq_T^2} = \int dx_1 dx_2 \, f_{a/h_1}(x_1, \mu_f^2) \, f_{b/h_2}(x_2, \mu_f^2) \, \frac{d\hat{\sigma}^F_{ab}}{dq_T^2}$$

- There is a hidden problem in $d\hat{\sigma}^F$ when $q_T \ll M$:

$$\int_0^{q_T^2} d\tilde{q}_T^2 \frac{d\hat{\sigma}^F_{ab}}{dq_T^2} \simeq \hat{\sigma}^{(0)}_{ab} \left[1 + \alpha_s \left(c_{12} \ln^2 \frac{M^2}{q_T^2} + c_{11} \ln \frac{M^2}{q_T^2} + c_{10} \right) + \ldots \right]$$

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Why resummation?

- In general, terms like $\sim \alpha_s^n \ln^{2n} \frac{M^2}{q_T^2} + \ldots$ will appear
- As soon as:

$$\alpha_s \ln^2 \left(M^2/q_T^2\right) \to 1$$

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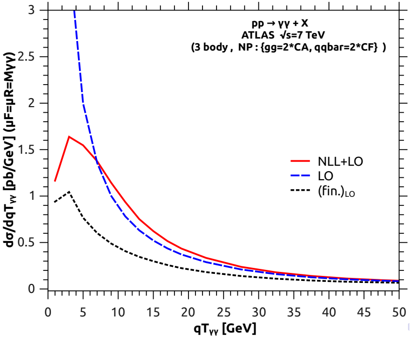

Summing logs to all orders

- Logs appear when integrating over QCD IR singularities (soft and collinear) and can be resummed via exponentiation (Sudakov-style, think QED)
  Need factorization of both dynamics and kinematics

- Dynamics factorization is a general feature of soft/collinear QCD emissions, schematically:

$$dw_n(q_1, \ldots, q_n) \simeq \frac{1}{n!} \prod_i dw_i(q_i) \qquad (\rightarrow \text{small } q_i)$$

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- In the q_T case, a trick is going to impact parameter space:

$$\int d^2 q_T \exp(-ib \cdot q_T)\delta(q_T - \sum_i q_{iT}) = \prod_i \exp(-ib \cdot q_{iT})$$

- Exponentiaton then holds in b-space, the big logs become

$$\log \frac{M^2}{q_T^2} \leftrightarrow \log M^2 b^2$$

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Summing logs to all orders

- I'll now jump straight to the final formula, but have a look at q_T resummation long history...

b-space formalism:
[Dokshitzer,Diakonov,Troian('78)] , [Parisi,Petronzio(79)] , [Kodaira,Trentadue(82)] , [Collins,Soper,Sterman(85)] , [Altarelli et al.(84)] , [Catani,dEmilio,Trentadue(88)] , [Catani,De Florian, Grazzini(01)] , [Catani,Grazzini(10)] , [Catani,Grazzini,Torre(14)] , [Catani,Cieri,De Florian,Ferrera,Grazzini('14)] .

In the framework of Effective Theories:
[Gao,Li,Liu(05)] , [Idilbi,Ji,Yuan(05)] , [Mantry,Petriello(10)] , [Becher, Neubert(10)] , [Echevarria,Idilbi,Scimemi(11)] .

In the context of transverse-momentum dependent factorization:
[DAlesio,Murgia(04)] , [Roger,Mulders(10)] , [Collins(11)] , [DAlesio,Echevarria,Melis,Scimemi(14)] , [Ceccopieri,Trentadue(14)] .

Effective q_T-resummation obtained with Parton Shower algorithms POWHEG/MC@NLO:
[Barzeetal.(12,13)] , [Hoeche,Li,Prestel(14)] , [Karlberg,Re,Zanderighi(14)] .

q_T resummation: Master Formula

- We only deal (here) with non-coloured final states

$$\frac{d\sigma^{F(res)}(s,q_T,M,y,\Omega)}{d^2q_T dM^2 dy d\Omega} = \frac{M^2}{s}\int \frac{d^2b}{(2\pi)^2} e^{ib\cdot q_T} S_c(M,b) \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} d\tilde{\sigma}_{c\bar{c}}^{F;h_1h_2\lambda_1\lambda_2}$$
$$\cdot\, C_{ca_1}^{h_1\lambda_1}(z_1,\alpha_s(b_0^2/b^2)) C_{\bar{c}a_2}^{h_2\lambda_2}(z_2,\alpha_s(b_0^2/b^2)) f_{a_1/h_1}(x_1/z_1,b_0^2/b^2) f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

with

$$S_c = \exp\left[-\int_{\frac{b_0^2}{b^2}}^{M^2} \frac{dq^2}{q^2}\left(A_c(\alpha_s(q^2))\log(M^2/q^2) + B_c(\alpha_s(q^2))\right)\right]$$

$$d\tilde{\sigma}_{c\bar{c}}^{F;h_1h_2\lambda_1\lambda_2} = d\hat{\sigma}_{c\bar{c}}^{F(0)} H_c^{F;h_1h_2\lambda_1\lambda_2}; \qquad x_{1,2} = \frac{M}{\sqrt{\hat{s}}}e^{\pm y}$$

$$K_{c,\dots}(\alpha_s) = \sum_n \left(\frac{\alpha_s}{\pi}\right)^n K_{c,\dots}^{(n)}$$

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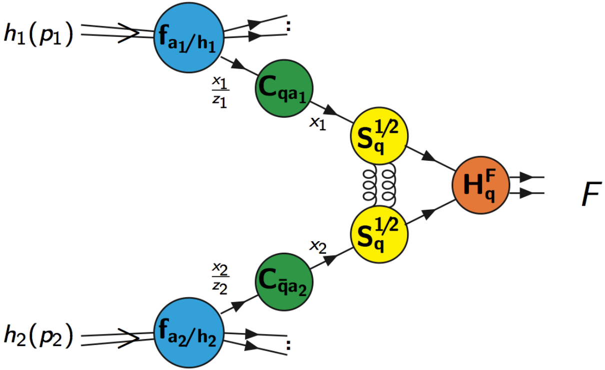

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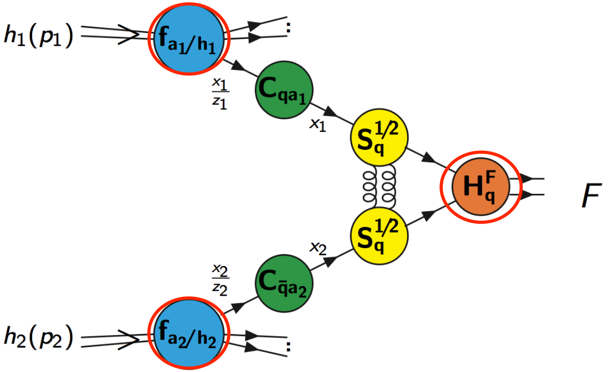


q_T resummation: Master Formula

- The formula is a small q_T approximation: it holds up to a formal $\mathcal{O}(q_T^2/M^2)$
- The formula is fully differential: even though the q_T spectrum is the main target, arbitrary observable distributions can be produced
- Most of the factors in master formula are universal: only the "modified partonic cross-section" is process-dependent

q_T resummation: Master Formula

- Universality is a consequence of factorization properties of QCD amplitudes on IR singularities
- All coefficients are perturbative: no new non-perturbative contribution beyond the PDFs
- In fact, all factors could be merged in generalized "b-dependent PDFs" were it not for the matter of spin correlations

Quarks, gluons and spin correlations

- The "generalized partonic cross-section" $d\tilde{\sigma}_{c\bar{c}}^{F;h_1 h_2 \lambda_1 \lambda_2}$ has (in general) a spin dependence
- To make this more explicit, look at the structure of H and C coefficients

$$C_{qa_i}^{\lambda_i h_i}(z_i, p_i, \mathbf{b}, \alpha_s) = C_{qa_i}(z_i, \alpha_s)\delta^{\lambda_i, h_i}$$

$$C_{ga_i}^{\lambda_i h_i}(z_i, p_i, \mathbf{b}, \alpha_s) = C_{ga_i}(z_i, \alpha_s)\delta^{\lambda_i, h_i} + G_{ga_i}(z_i, \alpha_s) D^{(\lambda_i)}(p_i, \mathbf{b})\delta^{\lambda_i, -h_i} \,, \quad i = 1, 2$$

$$D^{(\lambda_i)}(p_i, \mathbf{b}) = -e^{\pm 2\mathrm{i}\lambda_i(\varphi(\mathbf{b}) - \varphi_i)}$$

$$H_q^F = \frac{|\tilde{\mathcal{M}}_{q\bar{q}\to F}|^2}{|\mathcal{M}_{q\bar{q}\to F}^{(0)}|^2} \,, \qquad H_g^{F(h_1\lambda_1)(h_2\lambda_2)} = \frac{\left[\tilde{\mathcal{M}}_{gg\to F}^{h_1 h_2}\right]^* \tilde{\mathcal{M}}_{gg\to F}^{\lambda_1 \lambda_2}}{|\mathcal{M}_{gg\to F}^{(0)}|^2}$$

Quarks, gluons and spin correlations

- Here $\tilde{\mathcal{M}}^{h_1 h_2}_{ab\to F}$ is a IR-regulated helicity amplitude, obtained from the standard amplitude (UV-renormalized in $\overline{MS}$ and evaluated in dimensional regularization) via a subatraction operator

$$\tilde{\mathcal{M}}^{h_1 h_2}_{c\bar{c}\to F}(x_1 p_1, x_2 p_2, \Omega, \mu_R) = \left(1 - \tilde{I}_c(\epsilon, M^2, \mu_R)\right) \mathcal{M}^{h_1 h_2}_{c\bar{c}\to F}(x_1 p_1, x_2 p_2, \Omega, \mu_R, \epsilon)$$

  which removes remaning IR divergences

- This way, the formula "knows" about virtual corrections to the process to the order desired

Where is the resummation?

- We talked about "resummation", but where exactly do you resum?
- Look at the Sudakov term, use α_s evolution

$$\alpha_s(q^2) = \frac{\alpha_s(M^2)}{l} - \left(\frac{\alpha_s(M^2)}{l}\right)^2 \frac{\beta_1}{\beta_0} \log l + \dots \qquad (l = 1 + \beta_0 \alpha_s(M^2) \log(q^2/M^2))$$

$$- \int_{b_0^2/b^2}^{M^2} \frac{\mathrm{d}q^2}{q^2} \left[A_a(\alpha_s(q^2)) \log \frac{M^2}{q^2} + B_a(\alpha_s(q^2)) \right]$$

$$= \left(\frac{\alpha_s(M^2)}{\pi}\right)^{-1} \bar{g}^{(1)} + \left(\frac{\alpha_s(M^2)}{\pi}\right)^{0} \bar{g}^{(2)} + \left(\frac{\alpha_s(M^2)}{\pi}\right)^{1} \bar{g}^{(3)} + \dots$$

Where is the resummation?

$$\bar{g}^{(1)} = \frac{A^{(1)}}{\beta_0}\frac{\lambda + \log(1-\lambda)}{\lambda}$$

$$\bar{g}^{(2)} = \frac{B^{(1)}}{\beta_0}\log(1-\lambda) - \frac{A^{(2)}}{\beta_0^2}\left(\frac{\lambda}{1-\lambda} + \log(1-\lambda)\right) + \frac{A^{(1)}\beta_1}{\beta_0^3}\left(\frac{1}{2}\log^2(1-\lambda) + \frac{\log(1-\lambda)}{1-\lambda} + \frac{\lambda}{1-\lambda}\right)$$

$$\bar{g}^{(3)} = -\frac{A^{(3)}}{2\beta_0^2}\frac{\lambda^2}{1-\lambda} - \frac{B^{(2)}}{\beta_0}\frac{\lambda}{1-\lambda} + \frac{A^{(2)}\beta_1}{\beta_0^3}\left(\frac{\lambda(3\lambda-2)}{2(1-\lambda)^2} - \frac{(1-2\lambda)\log(1-\lambda)}{(1-\lambda)^2}\right) + \frac{B^{(1)}\beta_1}{\beta_0^2}\left(\frac{\lambda}{1-\lambda} + \frac{\log(1-\lambda)}{1-\lambda}\right) + A^{(1)}\left(\frac{\beta_1^2}{2\beta_0^4}\frac{1-2\lambda}{(1-\lambda)^2}\log^2(1-\lambda) + \log(1-\lambda)\left(\frac{\beta_0\beta_2-\beta_1^2}{\beta_0^4} + \frac{\beta_1^2}{\beta_0^4(1-\lambda)}\right)\right)$$

$$\lambda = \frac{1}{\pi}\beta_0\alpha_s(M^2)\log(M^2b^2/b_0^2)$$

- More contributions with the same structure from

$$C_{qa}(z, \alpha_s(b_0^2/b^2)) = C_{qa}(z, \alpha_s(M^2)) \exp\left[\int_{b_0^2/b^2}^{M^2} \frac{\mathrm{d}q^2}{q^2} \beta(\alpha_s(q^2)) \frac{\mathrm{d}\log(C_{qa}(z, \alpha_s(q^2)))}{\mathrm{d}\log(\alpha_s(q^2))}\right]$$

- ...and from PDF evolution, which uses the kernel solution to Altarelli-Parisi equation

$$f_{a/h}(x, b_0^2/b^2) = \int_x^1 \mathrm{d}z\, U_{ab}(x/z; b_0^2/b^2, \mu_F^2) f_{b/h}(z, \mu_F^2)$$

Resummation scale

- The large logarithms we resum can be redefined:

$$\log(M^2 b^2) = \log(\mu^2 b^2) + \log(M^2/\mu^2), \quad \mu \sim M$$

introducing an uncertainty in the resummation procedure

- This is dealt with by introducing a resummation scale (μ_{res}) and splitting all Sudakov-like integrals

$$\int_{b_0^2/b^2}^{M^2} \frac{\mathrm{d}q^2}{q^2} K(\alpha_s(q^2)) = \int_{b_0^2/b^2}^{\mu_{res}^2} \frac{\mathrm{d}q^2}{q^2} K(\alpha_s(q^2)) + \int_{\mu_{res}}^{M^2} \frac{\mathrm{d}q^2}{q^2} K(\alpha_s(q^2))$$

- Only the first term needs resummation: the second has no large logs. μ_{res} (similar to μ_R, μ_F) can be varied to estimate the uncertainty

4. reSolve

reSolve – What's this?

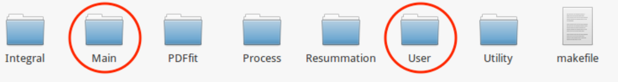
- The reSolve code is a MonteCarlo implementation of the *b*-space transverse momentum resummation formalism
- Still in its β version (upgrade out soon!): as of now just contain one process (the $\gamma\gamma$ SM background – soon DY, Higgs signal + interference) as a proof of concept
- Initial work on reSolve based on 2gres, non-public member of the XXres program family (Hres, DYres, . . .)

reSolve – Motivations

- Part of the motivation is historical: re-implementation, bug-checking
- The *b*-space resummation formula really lends itself to a general implementation, which is (for now) missing
- Then there are some "philosophical" choices: modularity, extendibility, transparency, parallelizability

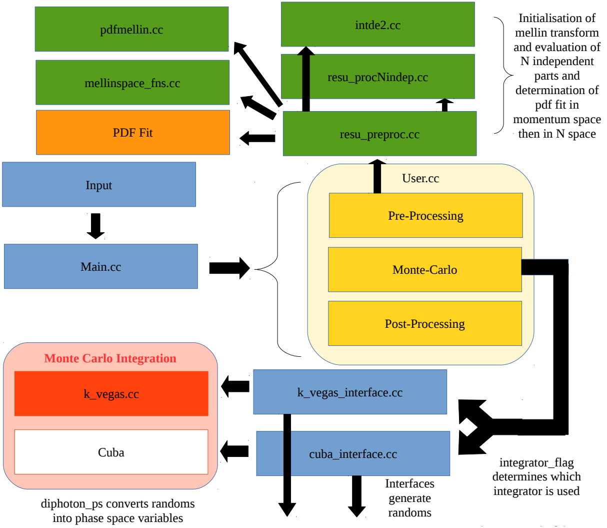
Modularity and program structure

- The program components

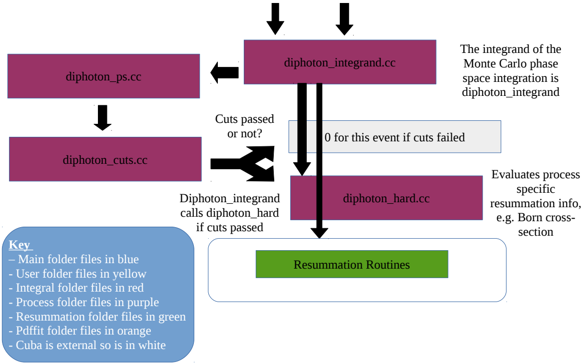


- These are as much as possible independent (they can be compiled and used separately)
- For transparency, an effort was made to keep the code clean and commented, and explaining everything the manual

How does this work? Structure I



How does this work? Structure II

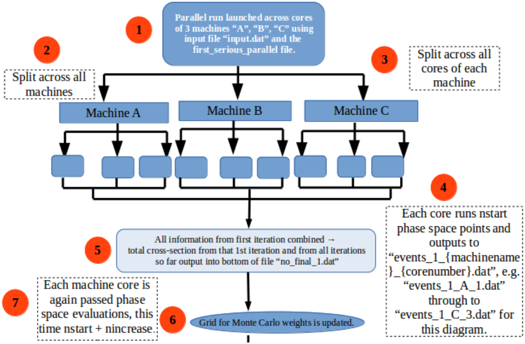


Integration and parallelization

- In keeping with the basic modular philosophy, we did not commit to a single MonteCarlo integrator
- Currently, two different integrators available: the public CUBA library and a custom VEGAS implementation dubbed k_vegas
- The CUBA library (if one wants to use it) must be downloaded and installed separately; the code can also run without it. Nicest CUBA feature is automatic parallelization for multi-core machines.

Integration and parallelization

- k_vegas is a C++ rewriting of the original LePage's VEGAS. It has been explicitly written to allow massive parallelization (over clusters or multiple Desktop machines) – this currently requires some work on part of the user



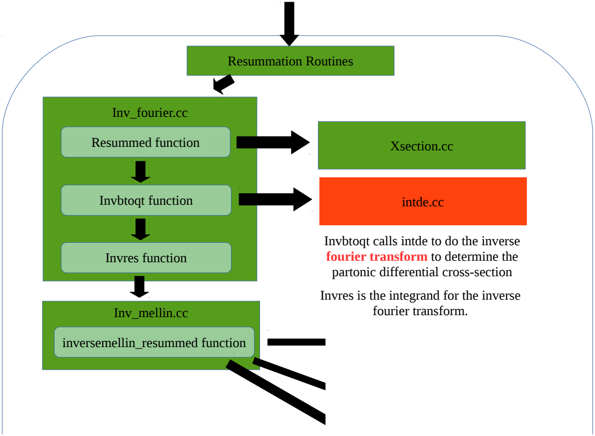
reSolve and resummation

- I will now give some more details about resummation implementation in reSolve .
- Keep in mind the master formula:

$$\frac{d\sigma^{F(res)}(s,q_T,M,y,\Omega)}{d^2q_T dM^2 dy d\Omega} = \frac{M^2}{s}\int \frac{d^2b}{(2\pi)^2} e^{ib\cdot q_T} S_c(M,b) \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} d\tilde{\sigma}^{F;h_1 h_2 \lambda_1 \lambda_2}_{c\bar{c}}$$

$$\cdot\, C^{h_1\lambda_1}_{ca_1}(z_1,\alpha_s(b_0^2/b^2)) C^{h_2\lambda_2}_{\bar{c}a_2}(z_2,\alpha_s(b_0^2/b^2)) f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

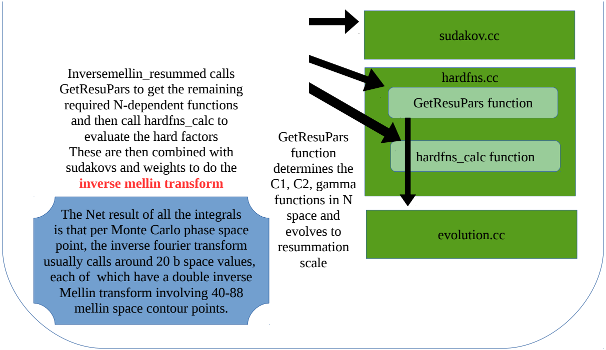
reSolve and resummation: flowchart I



Invbtoqt calls intde to do the inverse **fourier transform** to determine the partonic differential cross-section

Invres is the integrand for the inverse fourier transform.

reSolve and resummation: flowchart II



Mellin space and PDF fit

- The double convolution

$$\int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} C_{ca_1}^{h_1\lambda_1}(z_1, \alpha_s(b_0^2/b^2)) C_{\bar{c}a_2}^{h_2\lambda_2}(z_2, \alpha_s(b_0^2/b^2)) f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

  is easier to deal with by going to Mellin space

$$K_N = \int_0^1 \mathrm{d}z \, z^{N-1} K(z)$$

  which turns the convolutions in simple products and allows a simpler separation of the various scales b_0^2/b^2, μ_{res}, M.

- Downside: you need to fit the PDFs to an analytic form to define their Mellin transforms. Currently one of the weakest points of the code.

b integral and nonperturbative contributions

- The b integral (inverse F.T.) is one of the trickiest parts of the calculation
- The b-dependent functions are singular both at high and low b
- The low-b singularity is not physically meaningful, as small $b \Rightarrow$ high q_T where the formula breaks down anyway. We deal with this via the replacement

$$\log\left(\frac{\mu_S^2 b^2}{b_0^2}\right) \to \log\left(\frac{\mu_S^2 b^2}{b_0^2} + 1\right)$$

b integral and nonperturbative contributions

- The high-b singularity is trickier: functions become singular as

$$b^2 \rightarrow b_L^2 = \frac{b^2}{\mu_S^2} \exp\left(\frac{\pi}{\beta_0 \alpha_s(\mu_S^2)}\right) ,$$

which corresponds to $b_L \sim 1/\Lambda_{QCD}$: this is a manifestation of the QCD Landau pole! It is dealt with using

$$b \rightarrow b_* = \frac{b}{\sqrt{1 + b^2/b_{lim}^2}}$$

$$\text{with} \quad b_{lim} = \frac{b_0^{'}}{q} \exp\left(\frac{1}{2\alpha_s \beta_0}\right) , \quad b_0^{'} = 2 \exp\left[\gamma \frac{q}{\mu_S}\right]$$

b integral and nonperturbative contributions

- However this singularity and its regularization are not harmless. They signal the onset of nonperturbative (NP) effects which are not controlled by resummation.
- This gives an additional uncertainty to the resummation formula. This is estimated by adding a naive model for the NP effects:

$$S_{NP} = \exp\left(-g_{NP}^{c} b^2\right)$$

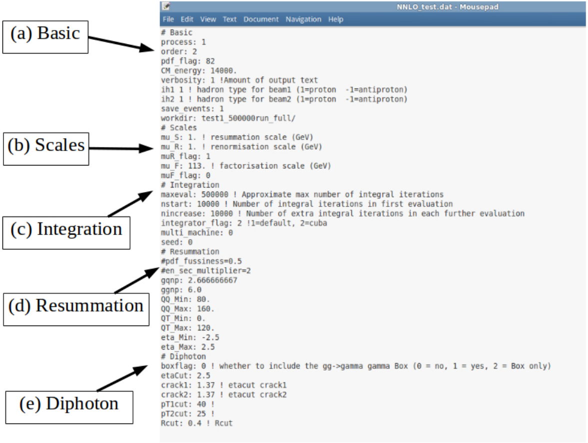
  and varying the g_{NP} coefficients in order to estimate their impact.

reSolve – Using the code

- Download and installation is straightforward: get the code from GitHub (https://github.com/fkhorad/reSolve), go to main dir and make
- All code (except for parts of CUBA) is C++11-standard complaint. Should be extremely portable; has been tested on multiple Linux distributions and MacOsX
- The code is simply run by writing down an input data card (text file) and running `./reSolve INPUT_FILE`

reSolve – input and output

- Sample input file

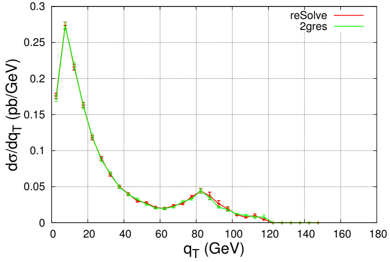


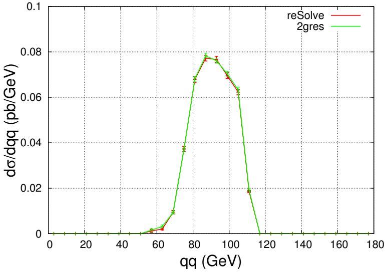
reSolve – input and output

- The main output (along the total value of the integration, χ^2 and error) are weighted events. They can be produced either in a minimal custom format or in an LHE-like style.
- Distributions of arbitrary observables can also be done "on the fly" or in a second moment using stored events
- Very easy to add more observables

Validation

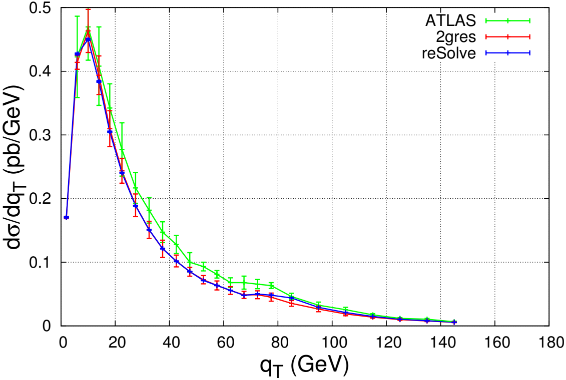
- Validation: sample comparison with 2gres





Validation

- Validation: comparison with data (includes matching with 2gNNLO)



Validation: matching

- A full data comparison typically requires matching with a fixed order (non-resummed) calculation to cover the region of $q_T \sim M$
- There are various possible strategies for matching; since `reSolve` doesn't implement any right now, I will skip the details. The strategy typically used in the b-space formalism can by found for instance in [Catani,Cieri,De Florian,Ferrera,Grazzini('14)]

reSolve – major planned improvements

- Addition of more processes: Higgs production, Drell-Yan, and Higgs signal-background interference in the $\gamma\gamma$ channel (underway)
- Better automatization of parallelization with k_vegas (underway)
- Production of events fully complying with the LHE accord (only partially implemented at the moment)
- Implementation of matching (which also entails the inclusion of fixed-order matrix elements)

Conclusions

- Precision calculations for processes at hadron colliders are one of the important challenges which lie ahead the theoretical particle physics community
- We must have the right tools to face the challenge: I hope `reSolve` will be a nice addition to the phenomenologist's toolbox