Queen Mary, University of London
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Intercollegiate Postgraduate Course in Elementary Particle Physics

Examination Paper 2: The Standard Model

Thursday, 26 January, 2006

Time allowed for the Examination: 3 hours.

Answer *ALL* of the 19 questions (60 marks total).

Books and notes may be consulted.

1. Explain how the E. M. interaction may be considered to arise from the requirement of local gauge invariance, basing your explanation on a wave function satisfying the Dirac equation. [2 marks]

2. The four momenta (k, p) and (k', p') describe the initial and final state kinematics of $\gamma\ e^-$ Compton scattering.

Draw the two Feynman diagrams for this process, and hence write down the first order matrix elements. [4 marks]

3. The cross section for the Compton scattering of a virtual photon with four momentum $k = q$ $(q^2 = -Q^2)$, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\hat{s}}\left[-\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}} + \frac{2\hat{t}Q^2}{\hat{s}\hat{u}}\right]$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables for $\gamma\ e^-$ Compton scattering and α is the fine structure constant.

What is the origin of the Q^2 dependent term? [1 mark]

Modify the above so that the cross section is that for the QCD Compton process. [4 marks]

4. Define, in terms of the usual four vectors, the scaling variable x for deep inelastic lepton - nucleon scattering (DIS) and give a simple interpretation in terms of the parton model. [2 marks]

By considering the photon-quark CMS frame explain briefly how scaling violation arises and indicate the basis for the DGLAP evolution equations. [4 marks]

5. Obtain an expression for the pseudo rapidity variable η in terms of the polar angle and describe the k_t clustering algorithm for jet finding. [3 marks]

6. How is the charged weak current expressed in the form

$$J^{+\mu} = \frac{G_F}{\sqrt{2}}\overline{u_\nu}\gamma^\mu(1-\gamma^5)u_e$$

found to be modified in neutron or nuclear β decay? [1 mark]

7. Draw Feynman diagrams for double β decay and neutrinoless double β decay. [1 mark]

What are the necessary conditions for neutrinoless double β decay to occur? [1 mark]

8. Describe the 'seasaw' mechanism and obtain the mass eigenvalues. [3 marks]

9. Show that electron neutrinos (ν_e) experience an additional effective potential V_{eff} in matter given by

$$V_{eff} = \sqrt{2} G_F N_e$$

where N_e is the electron density in matter. (You can use without proof the Fierz transformation.) [3 marks]

10. If the propagation of the neutrino mass eigenstates of energies E_1 and E_2 where $E^2 = p^2 + m^2$ and $m_1, m_2 << p$ is determined by a Schroedinger like equation show that the propagation in vacuo is given by

$$i\frac{d}{dt}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \left[\begin{pmatrix} p & 0 \\ 0 & p \end{pmatrix} + \begin{pmatrix} m_1/2p^2 & 0 \\ 0 & m_2/2p^2 \end{pmatrix}\right]\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

[2 marks]

11. If

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix},$$

obtain an expression for

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

in matter in the same form as that in vacuo. Do not attempt to multiply out your expression. [3 marks]

12. A spin one (vector) massless field A^μ coupling to a current J^μ satisfies the partial differential equation

$$\frac{\partial^2 A^\mu}{\partial t^2} - \nabla^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = J^\mu$$

Modify the above to apply for a massive, virtual, spin one particle of mass M interacting with the current J^μ. [1 mark]

Obtain the propagator from the modified equation. [2 marks]

13. Show how the propagator of massive but unstable vector bosons may be modified to take account of the decay. [2 marks]

Write down the matrix element for W^+ decay. [1 mark]

14. Explain the structure of the $SU(2) \times U(1)$ electroweak theory in the massless limit, explaining as you do so, the concepts of weak isospin, weak hypercharge and local gauge invariance. [4 marks]

15. Write down the transformation of the W_i^μ field under local gauge transformation and the form of the self energy term. [2 marks]

Do the same for the gluon field. [2 marks]

16. Demonstrate that by means of a suitable symmetry breaking of a local $U(1)$ gauge symmetry of a complex scalar field, which describes charged scalar particles, that the EM field A^μ acquires mass. [3 marks]

17. Write down Weinberg's choice of the scalar field in $SU(2) \times U(1)$ electroweak theory before and after symmetry breaking. [1 mark]

18. Obtain the couplings of the Higgs to the W boson [4 marks]

19. Explain how fermion masses may be generated in the Weinberg-Salam model and how the Kobayashi - Maskawa matrix may be related to the quark mass matrices? [4 marks]