Growing superamplitudes 'organically'

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HEP Young Theorists' Forum

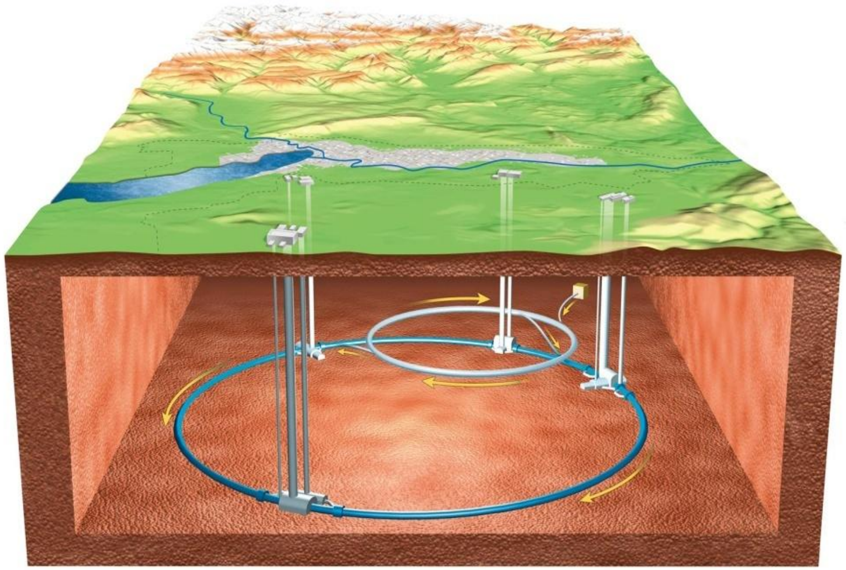
The experimental programme @ CERN

- Large Hadron Collider:

  New energies
  and hopefully
  'New Physics'.

- Possible 'New Physics':
  - Higgs particle
  - Supersymmetry
  - Extra dimensions
  - Strings
  - …

- How good are we in 'Old Physics'?



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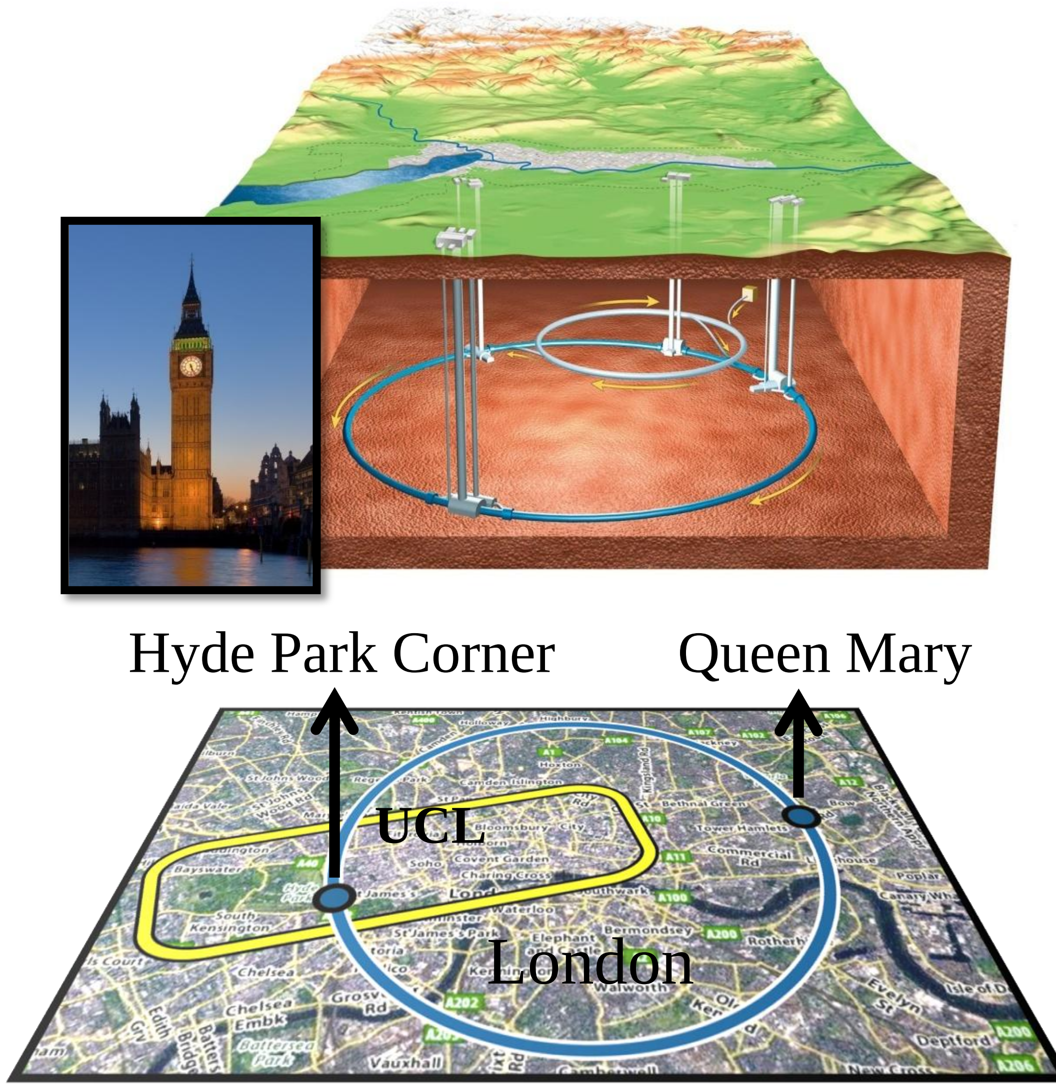

Yang-Mills theory

Yang-Mills

- Standard Model (Yang-Mills) @ LHC
  - Background/Benchmark processes.
  - Refined theoretical predictions needed to new experimental precision.
  - Get better in ‘Old Physics’ in order to be able to identify ‘New Physics’.

$$A_n(\{p_i, h_i, a_i\}) = 2^{n/2} g^{n-2}\,\mathrm{tr}\left[t^{a_1}\cdots t^{a_n}\right]\mathcal{A}_n\left(p_1, h_1; \ldots; p_n, h_n\right) + \text{perms} + \text{multi-tr}$$

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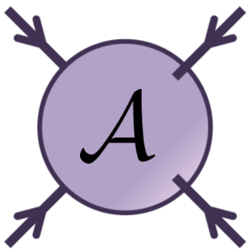

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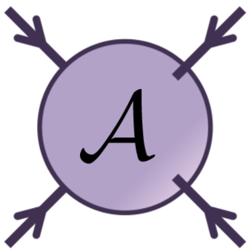

generators of nonabelian gauge group — momentum — helicity

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coupling constant — colour part — partial amplitude

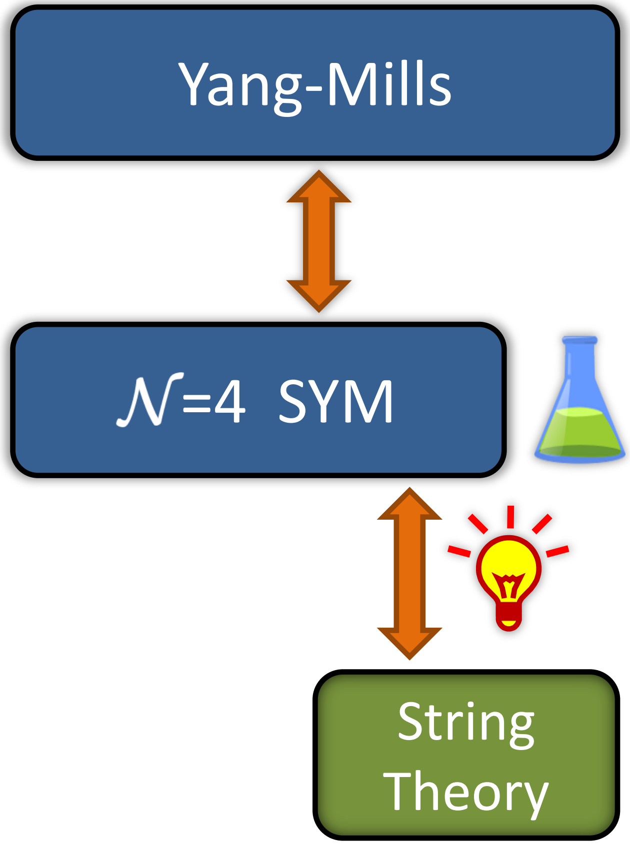
- Yang-Mills: partial amplitudes are colour-ordered

Maximally supersymmetric theories

- $\mathcal{N}$=4 Super-Yang-Mills:
  - Perfect Lab for testing new techniques.
  - AdS/CFT: Clues from and for String Theory.

Example: MHV amplitude / polygonal lightlike Wilson loop duality

[Alday, Maldacena] (2007) [Drummond, Korchemsky,Sokatchev, Henn; Brandhuber, Heslop, Travaglini]



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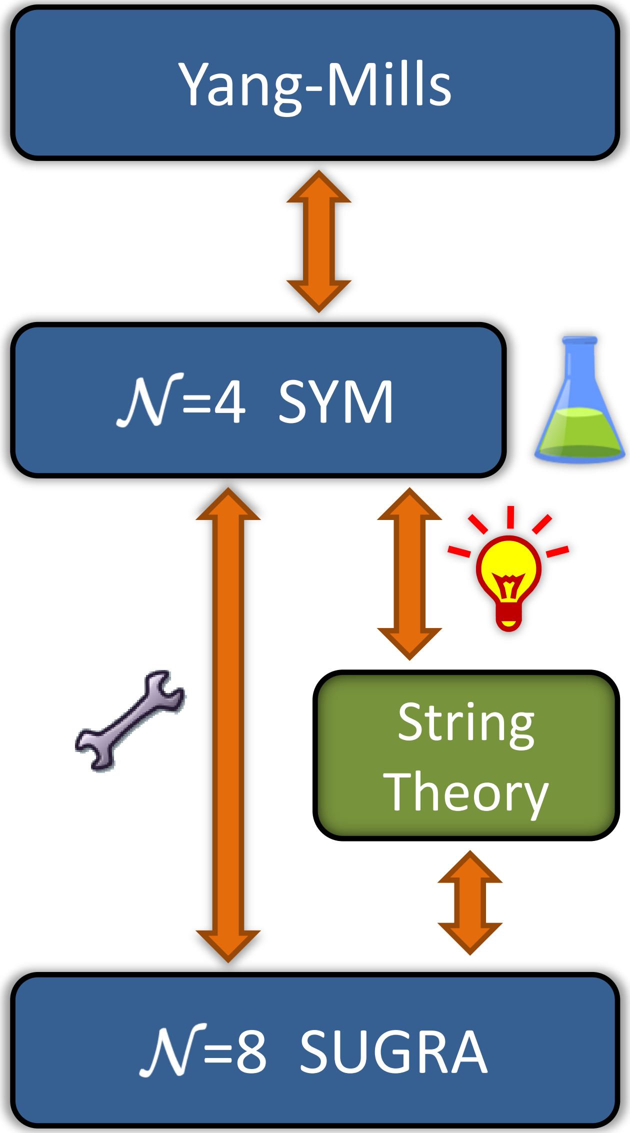
- $\mathcal{N}$=8 Supergravity:
  - Same techniques can be applied.
  - Relations to String Theory and $\mathcal{N}$=4 SYM.
  - Unexpected cancellations : UV finite?

[Bern, Dixon, Roiban. hep-th/0611086 (review)]

- The effect of Supersymmetry:

More complicated Langrangian but simpler amplitudes!

[Arkani-Hamed, Cachazo, Kaplan. 0808.1446 (review)]



On-shell superspace

- Massless momenta: $p_i^\mu \sigma_\mu^{a\dot{a}} = p_i^{a\dot{a}} = \lambda_i^a \tilde{\lambda}_i^{\dot{a}}$.
- Variables: $\{\lambda_i, \tilde{\lambda}_i, h_i\}$, for real momenta: $\overline{\lambda} = \pm\tilde{\lambda}$.
- Lorentz invariants: $\langle i\, j\rangle = \lambda_i^a \lambda_{ja}, \quad [i\, j] = \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_{j\dot{a}}.$ $\quad a = 1, 2$, $\dot{a} = 1, 2$
- Using Grassman (anticommuting) variables η_i :

$$\Phi(\lambda, \tilde{\lambda}, \eta) = G^+(\lambda, \tilde{\lambda}, \eta) + \eta^A \Gamma_A(\lambda, \tilde{\lambda}, \eta) + \frac{1}{2}\eta^A \eta^B S_{AB}(\lambda, \tilde{\lambda}, \eta)$$
$$+ \frac{1}{3!}\eta^A \eta^B \eta^C \epsilon_{ABCD} \overline{\Gamma}^D(\lambda, \tilde{\lambda}, \eta) + \frac{1}{3!}\eta^A \eta^B \eta^C \eta^D \epsilon_{ABCD} G^-(\lambda, \tilde{\lambda}, \eta)$$

[Nair] (1988)

$$A = 1, \ldots, \mathcal{N}$$

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helicity 1/2

scalars

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fermions

gluon

$A = 1, \ldots, \mathcal{N}$

$$\sum_{i=1}^{n} \lambda_i^a \tilde{\lambda}_i^{\dot{a}} = 0, \qquad \sum_{i=1}^{n} \lambda_{ia} \eta_i^A = 0.$$

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superfield; helicity 1/2; scalars

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fermions; gluon

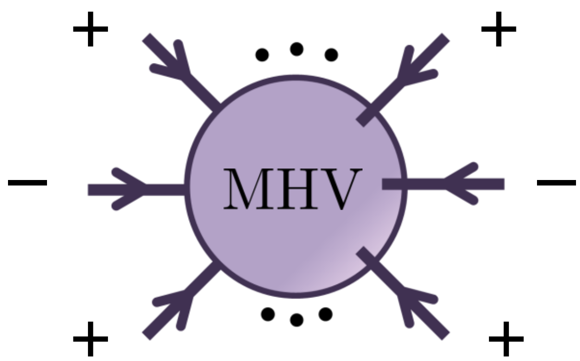
- Superamplitudes are defined on the surface: $A = 1, \ldots, \mathcal{N}$

$$\sum_{i=1}^{n} \lambda_i^a \tilde{\lambda}_i^{\dot{a}} = 0, \quad \sum_{i=1}^{n} \lambda_{ia} \eta_i^A = 0.$$

total momentum ← ; → total supercharge Q_a^A

MHV amplitudes

- Simplest non-vanishing amplitudes.
- Used as vertices to reformulate the perturbative expansion in terms of 'MHV Diagrams' with scalar propagators.



[Cachazo, Svrcek, Witten] (2004)

$$\mathcal{A}_{n;0}^{\text{MHV}} = i(2\pi)^4 \delta^{(4)}\Big(\sum_i \lambda_i^a \tilde{\lambda}_i^{\dot{a}}\Big) \frac{\langle r\ s\rangle^4}{\langle 1\ 2\rangle\langle 2\ 3\rangle \cdots \langle n\ 1\rangle}$$

[Parke-Taylor] (1986)

tree-level

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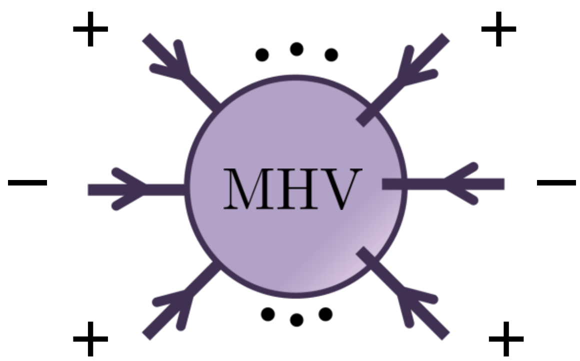

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momentum conservation

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All-in-one package

- Superamplitudes: amplitudes of superfields.

$$\mathcal{A}_n(\lambda_i, \tilde{\lambda}_i, \eta_i) = \mathcal{A}(\Phi_1 \ldots \Phi_n)$$

- General form of a superamplitude: [Drummond, Henn, Korchemsky, Sokatchev] (2008)

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expandable in η's

MHV, NMHV, N^2MHV, $\overline{\mathrm{MHV}}$

$$\mathcal{P}_n^{(0)} + \mathcal{P}_n^{(4)} + \mathcal{P}_n^{(8)} + \ldots + \mathcal{P}_n^{(4n-16)} = \mathcal{P}_n$$

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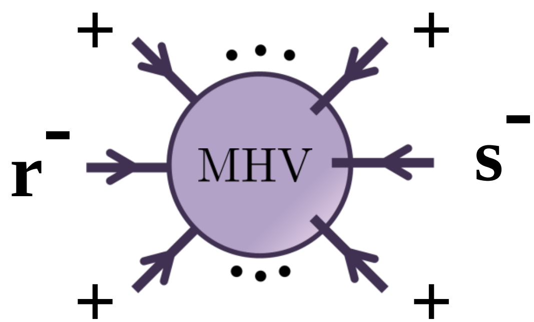
powers of η components

- Example: the part proportional to $(\eta_r^1 \ldots \eta_r^4)(\eta_s^1 \ldots \eta_s^4)$ (all gluons) $\longrightarrow$



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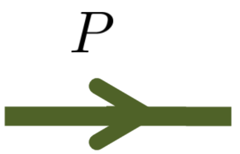
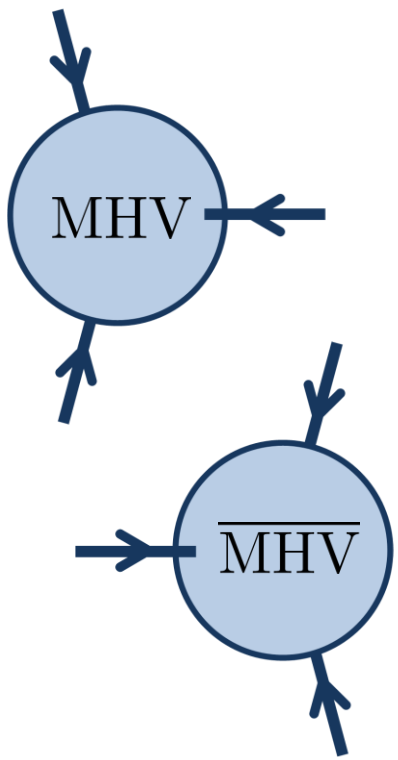
- Fairtrade: no one has to calculate 100’s of Feynman diagrams, or supergraphs.

‘Seeding’ superamplitudes

- Knowledge of just the analytic structure of the propagator in complexified Minkowski.
- Three-point amplitudes (non-vanishing for complex momenta).



$$\propto \frac{1}{P^2}$$



$$\mathcal{A}_{3;0}^{\text{MHV}} = \frac{\delta^{(8)}\left(\sum_{i=1}^{3} \lambda_i \eta_i\right)}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}$$

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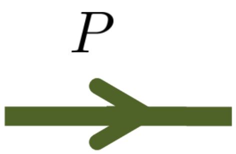

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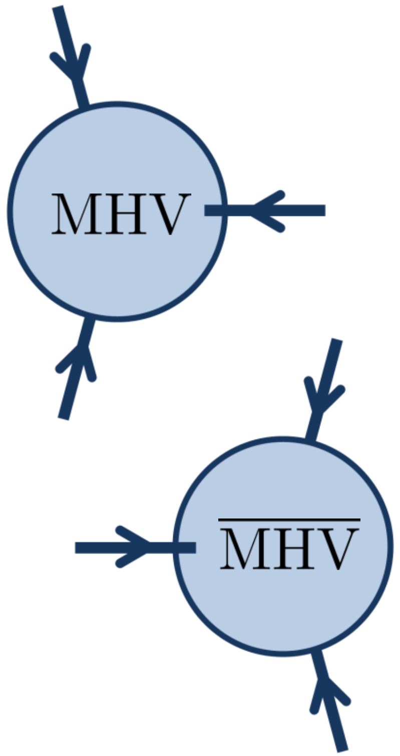

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- First clues of the pattern:

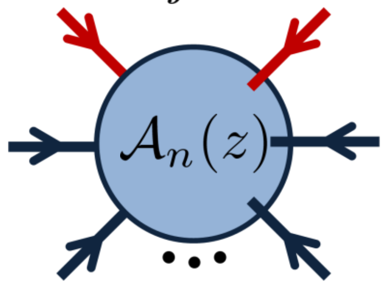
$\mathcal{N}$=8 SUGRA ~ ($\mathcal{N}$=4 SYM)^2

Shifted superamplitudes

$$\eta_i(z) = \eta_i + z\eta_j$$

$$\tilde{\lambda}_i(z) = \tilde{\lambda}_i + z\tilde{\lambda}_j \qquad \lambda_j(z) = \lambda_j - z\lambda_i$$

- Deformed amplitude $\mathcal{A}_n(z)$.
- On-shell quantity for all values of z,
- but with complex momenta.
- Overall momentum conserved,
- Overall supercharge conserved.



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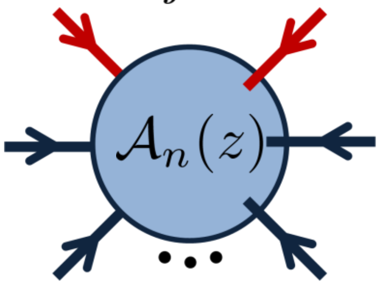

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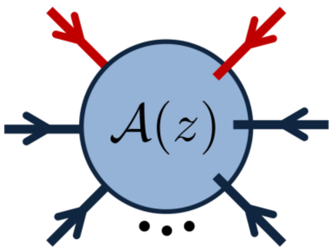
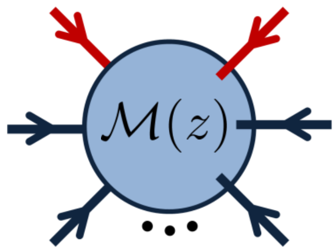
- As the deformation parameter $z \to \infty$ [Arkani-Hamed, Kaplan] (2008)



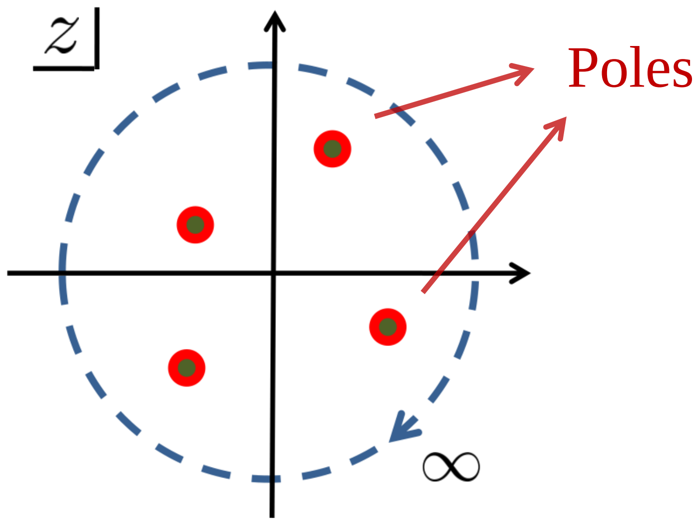
$\propto \frac{1}{z}$



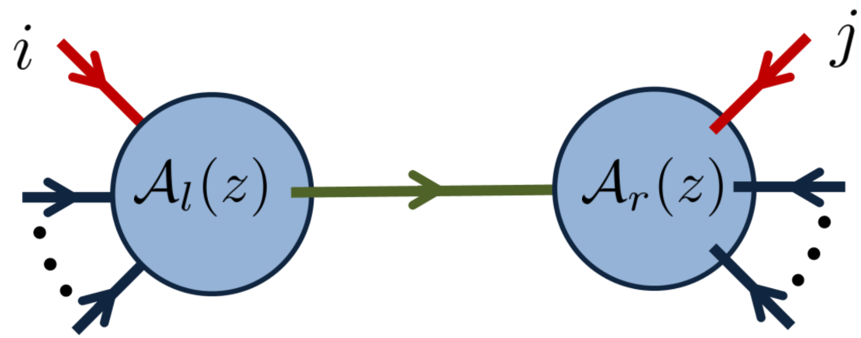
$\propto \frac{1}{z^2}$

- Amplitudes always vanish at infinity under supersymmetric shifts.
- Even better behaviour for gravity amplitudes due to cancellations.

On-shell recursion relations (BCFW)



$$\oint_\infty \frac{A^{\text{tree}}(z)}{z}\, dz = 0$$



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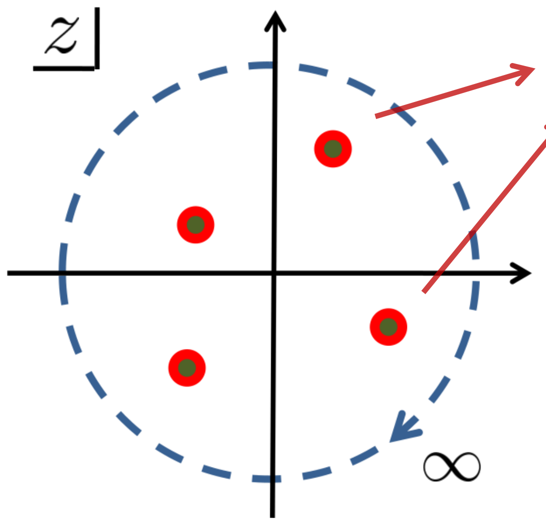

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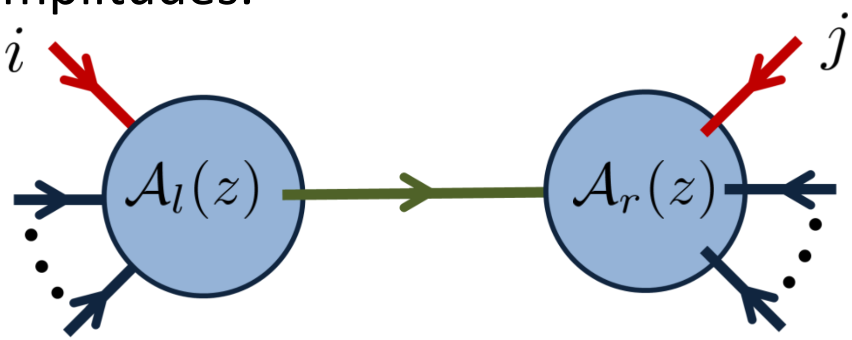
Cauchy's Theorem

$$\mathcal{A}^{\text{tree}}(0) + \sum_{\text{poles } z_p} \frac{\text{Res}[\mathcal{A}^{\text{tree}}(z)]|_{z_p}}{z_p} = 0$$

- Source of Poles: intermediate propagator going on-shell.
- Residues: products of lower point amplitudes.



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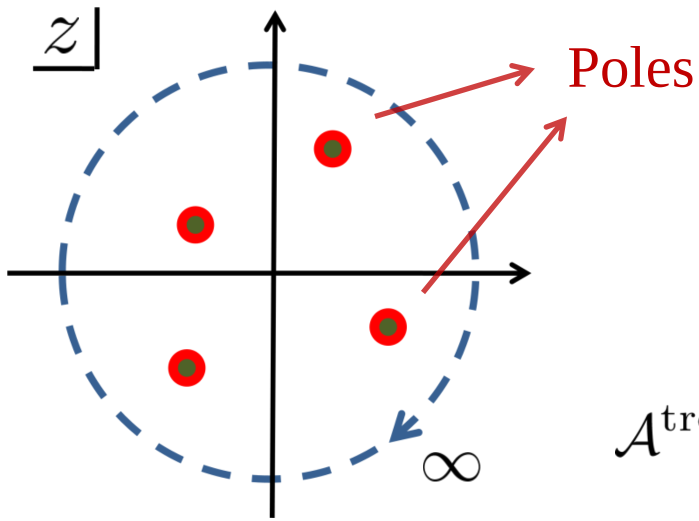

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[Britto, Cachazo, Feng +Witten] (2004-2005)
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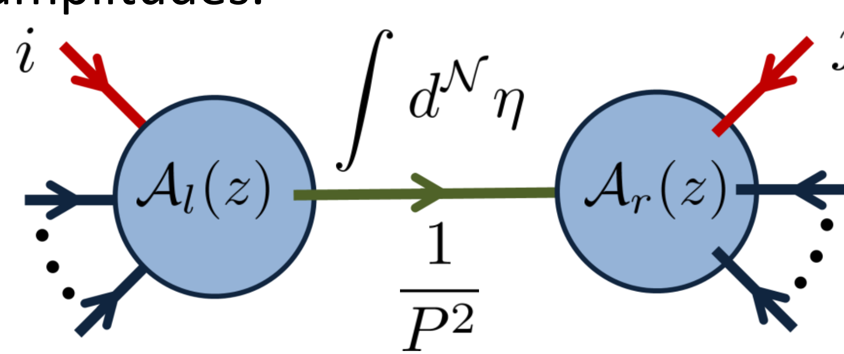
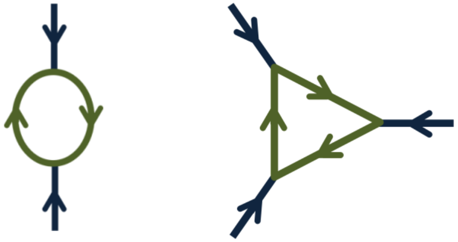
$$A_n = \sum_{z_p}$$



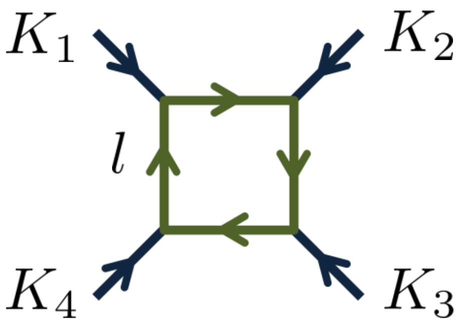
- Starting from 3-points we can calculate any tree superamplitude.

One-loop expansion

- 1-loop: integration over loop momentum l.
- 4D: amplitudes expandable in a known basis of integrals: boxes, triangles, bubbles + rational part.

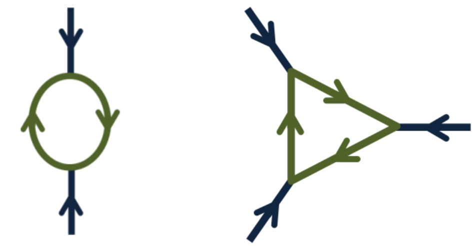


$$I_i(K_1, \ldots, K_i) = (4\pi)^{2-\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{1}{l^2 (l+K_1)^2 \cdots (l-K_i)^2}$$



One-loop expansion

- 1-loop: integration over loop momentum l.
- 4D: amplitudes expandable in a known basis of integrals: boxes, triangles, bubbles + rational part.



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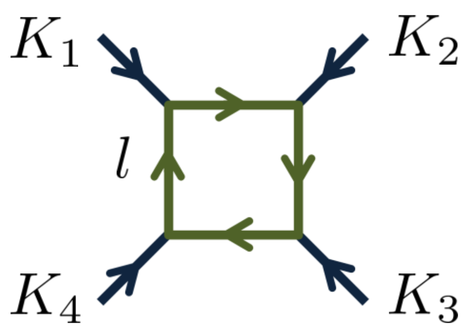
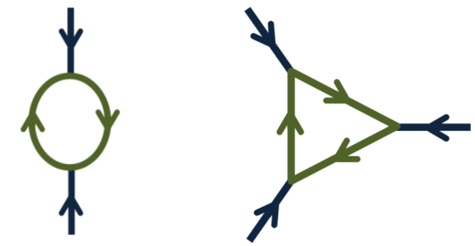
# of internal propagators

clusters of external momenta



$$\mathcal{A}_{n;1} = \sum_{\mathcal{P}(\{K_i\})} \mathcal{C}(K_1,K_2,K_3,K_4)\, I_4(K_1,K_2,K_3,K_4)$$

One-loop expansion



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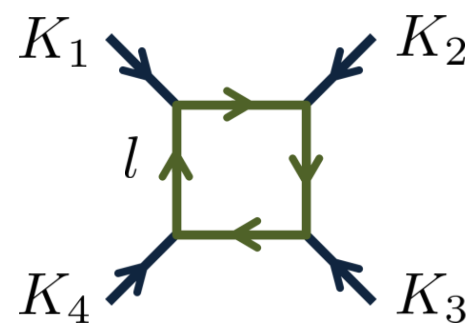
i → # of internal propagators

$K_1, \ldots, K_i$ → clusters of external momenta



- "No-triangle hypothesis": in maximal supersymmetry, amplitudes contain only boxes.

[Bern, Dixon, Dunbar, Kosower] (1994) [Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager] (2006) [Bjerrum-Bohr, Vanhove] (2008)

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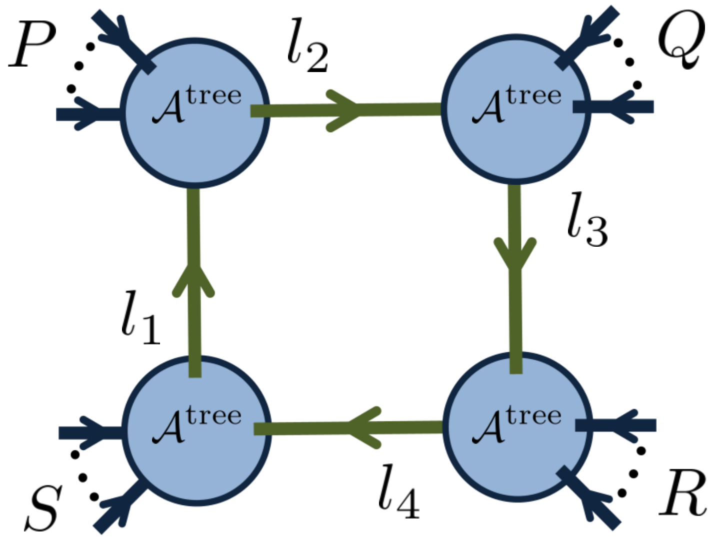
1 → one-loop

$\mathcal{P}(\{K_i\})$ → all different clusterings

$\mathcal{C}$ → rational functions of the particle spinors

Generalised unitarity (quadruple cuts)

- One-loop supercoefficients from trees.



$$= \mathcal{C}(P, Q, R, S)$$

[Britto, Cachazo, Feng] (2005)

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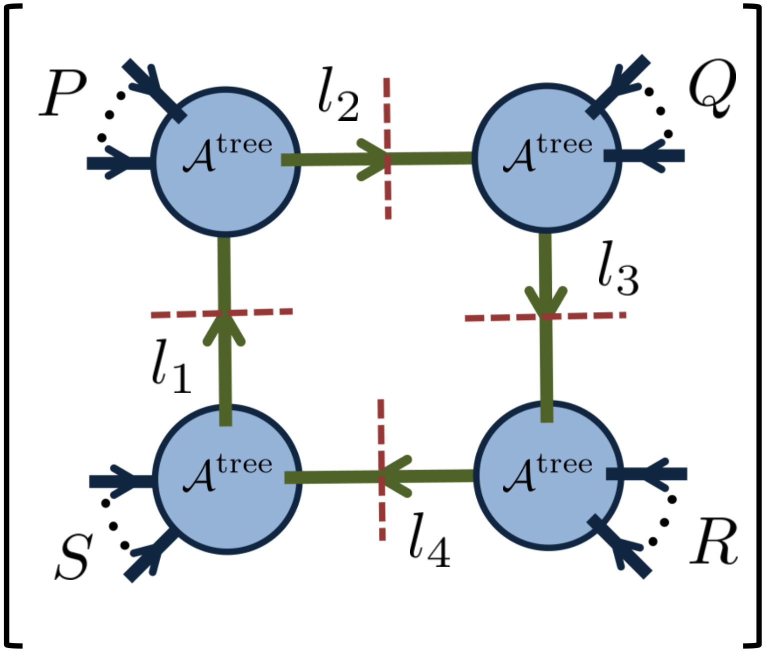

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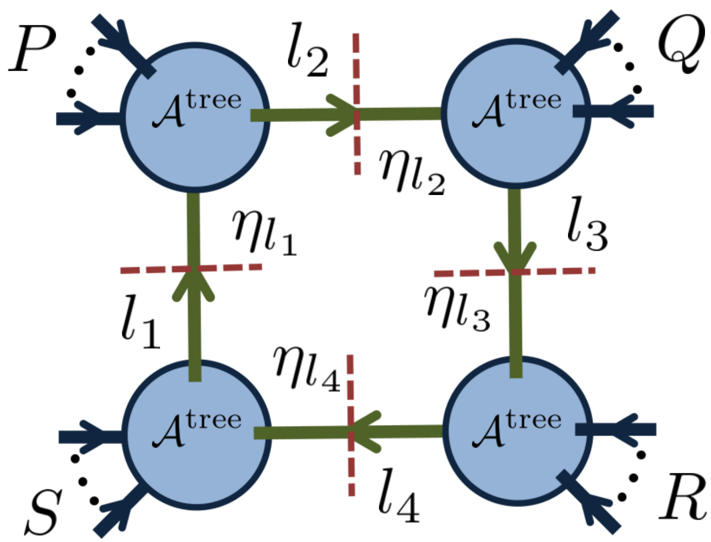
average over the two solutions $\{(l_i)^2 = 0\}$

[Britto, Cachazo, Feng] (2005)

Generalised unitarity (quadruple cuts)

- One-loop supercoefficients from trees.

$$\int \prod_{i=1}^{4} d^{\mathcal{N}} \eta_{l_i} \Big[\quad \Big] = \mathcal{C}(P, Q, R, S)$$



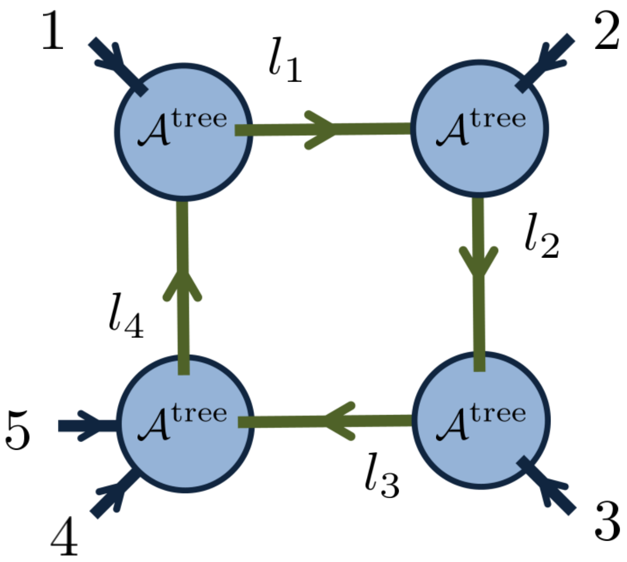
integration
=
differentiation

average over the two solutions $\{(l_i)^2 = 0\}$

[Britto, Cachazo, Feng] (2005) [Drummond, Henn, Korchemsky, Sokatchev] (2008)

- Supercoefficients from tree superamplitudes, without any integration!
- BCFW: any tree superamplitude ,
- Unitarity: all supercoefficients, therefore, all one-loop superamplitudes.

Example: 5-pt MHV SYM coefficients



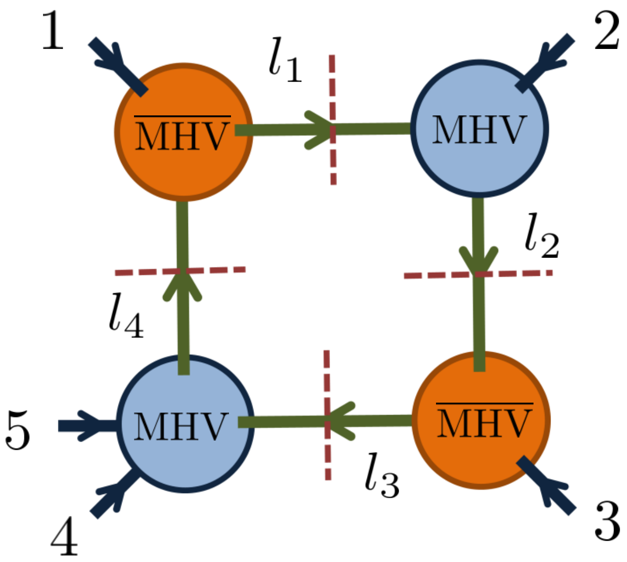
$$C_5^{1m}(1,2,3,45)$$

one massive corner

Example: 5-pt MHV SYM coefficients

- One contributing solution:

$$l_1 = z_1 \lambda_2 \tilde{\lambda}_1, \qquad l_2 = l_1 + p_2, \ \ldots$$



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- The quadruple cut diagram gives:

$$\frac{1}{2} \frac{\delta^{(4)}(\eta_{l_4}[1 l_1] + \eta_1 [l_1 l_4] + \eta l_1 [l_4 1])}{[l_4 1][1 l_1][l_1 l_4]}$$
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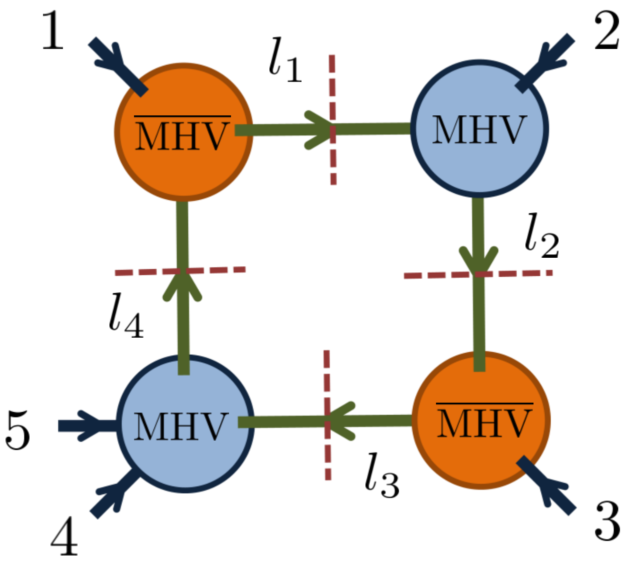
$$\times \frac{\delta^{(8)}(\lambda_{l_3}\eta_{l_3} + \lambda_4 \eta_4 + \lambda_5 \eta_5 - \lambda_{l_4} \eta l_4)}{\langle l_3 4\rangle \langle 45\rangle \langle 5 l_4\rangle \langle l_4 l_3\rangle}$$



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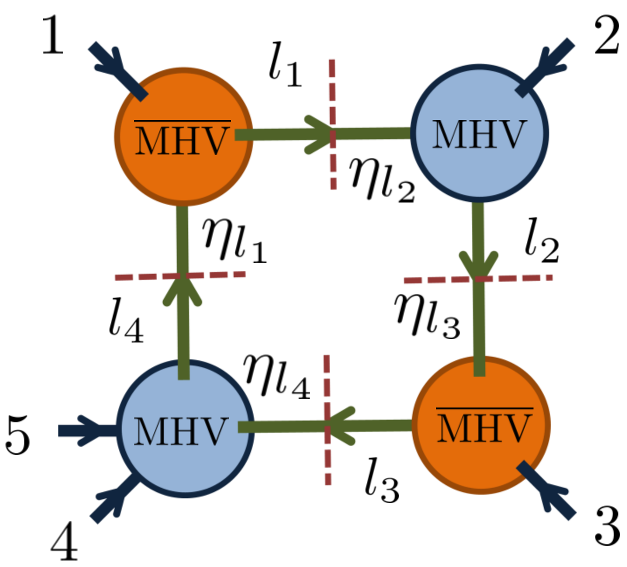

$$\frac{1}{2}\int \prod_{i=1}^{4} d^4\eta_{l_i} \frac{\delta^{(4)}(\eta_{l_4}[1l_1] + \eta_1[l_1 l_4] + \eta l_1[l_4 1])}{[l_4 1][1 l_1][l_1 l_4]}$$

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overall supercharge conservation

- Grassman integrations + supermomentum conservation:

$$C_5^{1m}(1,2,3,45) = \frac{1}{2}(p_1+p_2)^2 (p_2+p_3)^2 \frac{\delta^{(8)}(\sum_{i=1}^{5}\lambda_i\eta_i)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

one massive corner

s $\quad$ t

Relations between SYM and supergravity

- KLT relations heuristically derived from string theory. [Kawai, Lewellen, Tye] (1986)
- New relations from field theory:

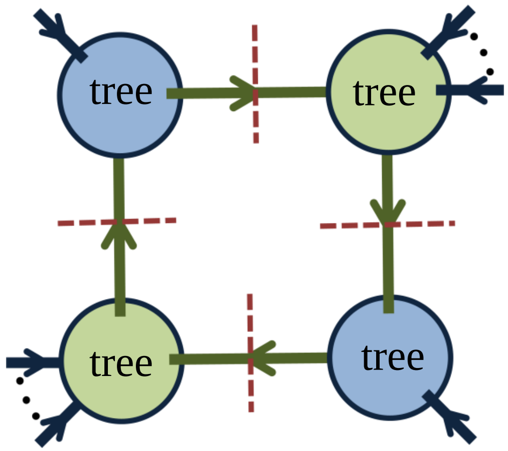
'dressing function'

$$\mathcal{M}_n^{\text{tree}}(1,\ldots,n) = \sum_{\mathcal{P}(2,\ldots,n-1)} \left[\mathcal{A}_n^{\text{tree}}(1,\ldots,n)\right]^2 \overbrace{G(1,\ldots,n)}$$

permutations ← $\mathcal{P}(2,\ldots,n-1)$

[Drummond, Spradlin, Volovich, Wen] (2009)

- Dressing factions calculated with on-shell recursion.
- 1^{st} and n^{th} leg not permuted,
- G independent of η_1 and η_n.



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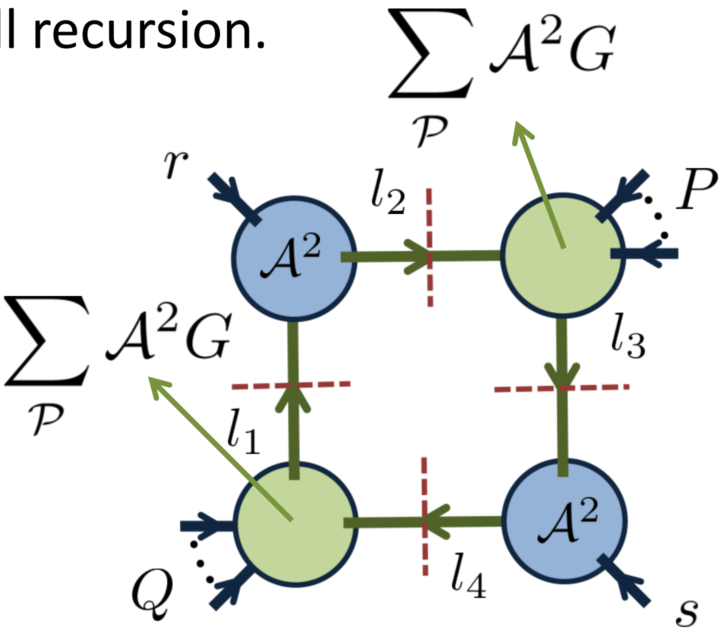
- Plugging them into unitarity we obtain relations between one-loop coefficients:

$$\mathcal{C}_{2me}^{\mathcal{N}=8}(r,P,s,Q) = \quad \text{(MHV case)}$$

$$\sum_{\mathcal{P}(\{P\})} \sum_{\mathcal{P}(\{Q\})} \left[\mathcal{C}_{2me}^{\mathcal{N}=4}(r,P,s,Q)\right]^2$$

$$\times\, 2\, G(-l_2,P,l_3)G(-l_4,Q,l_1)\Big|_{\text{cut sol.}}$$



[PK, Travaglini, Spence] (to appear)

Some open questions

- Existence of a full basis of integrals beyond one loop, so that we can better exploit on-shell methods for more loops.

- Relations between SYM and supergravity beyond one loop.

- Better understanding of cancellations in $\mathcal{N}$=8 supergravity.

- IS $\mathcal{N}$=8 supergravity UV finite?

  (If no, at what number of loops do the UV divergences appear?)

- Simpler results than intermediate expressions. Is there a shortcut? Possibly a new sort of dual formulation of the theory has yet to be discovered.

www.strings.ph.qmul.ac.uk/~pka/

Superamplitudes

Panagiotis Katsaroumpas, QMUL

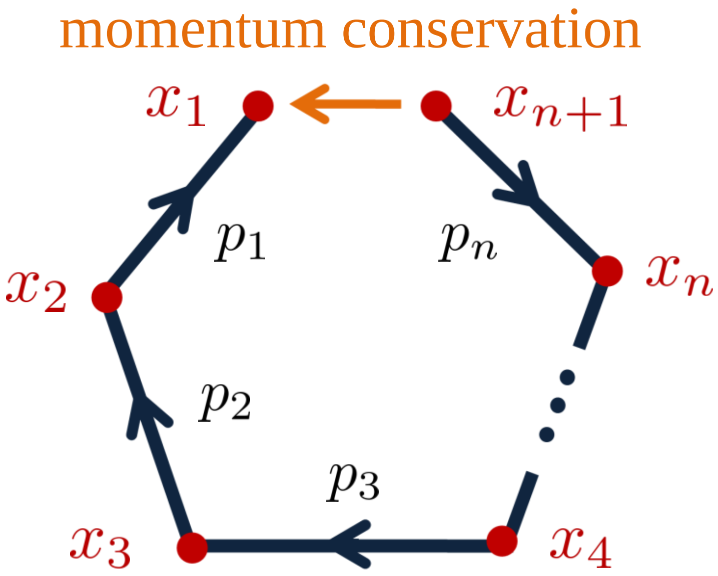
Cambridge, 16 April 2009
Beyond Part III, Young Researchers in Mathematics

The mysterious Dual space

- Dual x-space: redefinition of momentum space:

$$p_i := x_i - x_{i+1},$$

$$(\eta_i \lambda_i := \theta_i - \theta_{i+1}).$$



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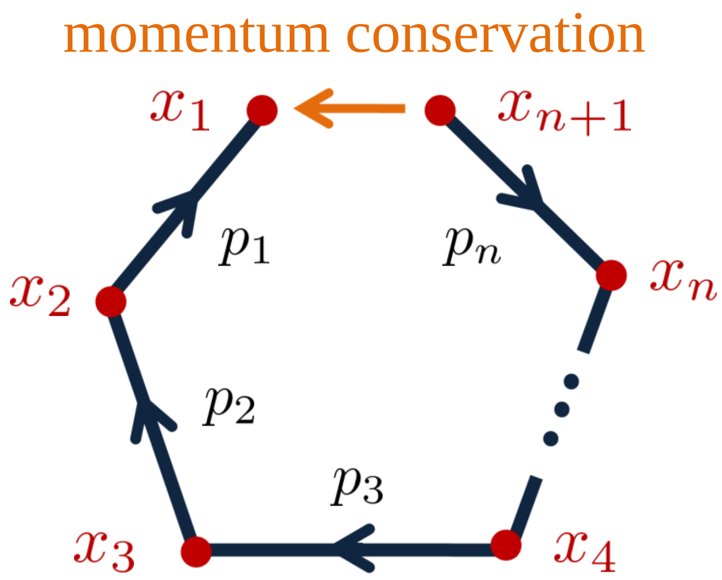
- In dual space (x, λ, θ), $\mathcal{N} = 4$ amplitudes are manifestly superconformally covariant.



- Tree MHV superamplitudes transform covariantly under inversions:

$$x^\mu \to \frac{x^\mu}{x^2} \qquad \mathcal{A}(1,2,\ldots,n) \to \mathcal{A}(1,2,\ldots,n) \prod_{k=1}^{n} x_k^2 .$$

- Property shown to hold for any tree superamplitude using on-shell recursion relations.

[Brandhuber, Heslop, Travaglini] (2008)

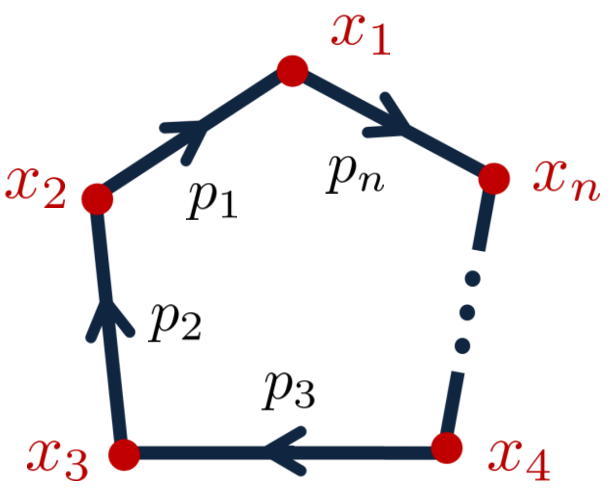
- One-loop supercoefficients shown to transform covariantly using quadruple cuts.

MHV amplitude / Wilson loop duality

- For MHV amplitudes of gluons, the factor

$$\frac{\mathcal{A}_n^{(L)}(p_1, p_2, \ldots, p_n)}{\mathcal{A}_n^{\text{tree}}(p_1, p_2, \ldots, p_n)}$$

is a helicity-blind kinematical function at any loop.



$$\mathcal{W}[\mathcal{C}_n] = \text{tr}\ \mathcal{P}\ \exp\left[ig \oint_{\mathcal{C}_n} dx^\mu A_\mu^a(x)\, t^a\right].$$

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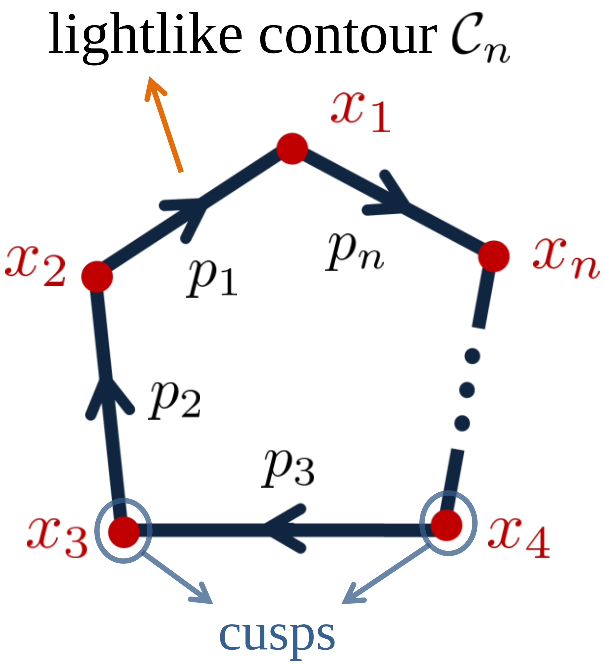

- Its finite part is equal to that of the expectation value of a polygonal lightlike Wilson loop living in the dual space:

path-ordered ... Expandable in g

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[Drummond, Korchemsky, Sokatchev, Henn] (2007)
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- Integration in the vicinity of the cusps produces UV divergences
- UV divergences match the IR divergences of the amplitude.